INF539: refresher basic data structures in C

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Plan

Good reading: Cormen/Leiserson/Rivest *Introduction to Algorithms*; also, X lecture notes (using Java) for INF361, INF411, INF421 (theoretical).

I. Arrays

Z) Warming up

```c
#include <stdio.h>

int main(int argc, char *argv[]){
    int a[10];
    int i;

    for(i = 0; i < 10; i++)
       a[i] = i;
    printf("a =");
    for(i = 0; i < 10; i++)
       printf(" %d", a[i]);
    printf("\n");
    return 0;
}
```

Array: dynamic allocation

```c
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char *argv[]){
    int *a = (int *)malloc(10 * sizeof(int));
    int i;

    for(i = 0; i < 10; i++)
       a[i] = i;
    printf("a =");
    for(i = 0; i < 10; i++)
       printf(" %d", a[i]);
    printf("\n");
    free(a);
    return 0;
}
```


A) Insertion sort

Principle: we suppose that `t[0..i[` is already sorted and we insert `t[i]` where it belongs.

t	9	2	5	13	4	2	6	8	10

Program:

```c
static void insertionSort(int[] t, int n){
  for(int i = 1; i < n; i++){
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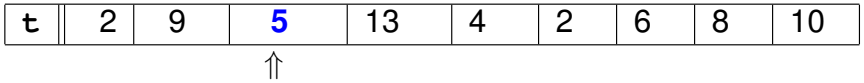

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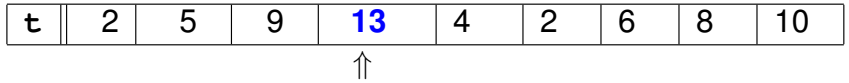

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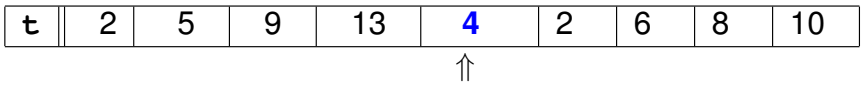

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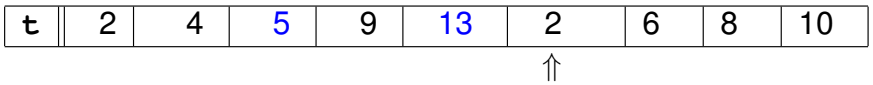

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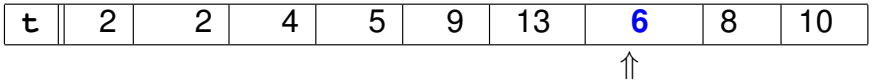

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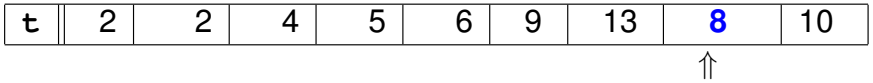

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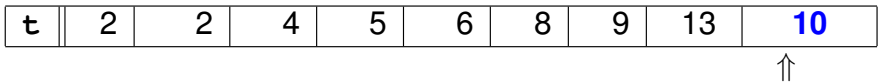

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}
```

The program (2/3)

```
// t[0..i-1] is sorted
// look for the place of t[i] defined by
// j s.t. t[j] <= t[i] < t[j+1]
static int findPlace(int[] t, int i){
  int j;

  for(j = i-1; j >= 0; j--)
    if(t[j] <= t[i])
      break;
  // if j < 0, t[i] < t[0], we return 0
  // if t[j] <= t[i] < t[j+1], we return j+1
  return (j+1);
}
```


The program (3/3)

```
static void insert(int[] t, int i, int j){
  int tmp = t[i], k; // copy

  // t[0..j-1][j..i-1] -> t[0..j-1][][j+1..i]
  for(k = i; k > j; k--)
    t[k] = t[k-1];
  t[j] = tmp;
}
```

Rem. We can find a more compact code using careful programming.

Rem. See `man qsort` for programs using the `libc` version of quicksort.

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B) Eratosthenes and beyond

Eratosthenes: how do we enumerate prime numbers in $[2, X]$?

It is much easier to enumerate composite numbers and then deduce the primes as being not composite.

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Prop. A composite number C has a prime factor $\leq \sqrt{C}$.

- empty sets denote primes;
- non-empty sets contain the prime factors of the composite number.

Eratosthenes: algorithm

Sieve:

1. Set $T[x] = \emptyset$ for $x \in [2,X]$;
2. **for** $p = 2$ **to** $\lfloor \sqrt{X} \rfloor$ **do**
   if $T[p] = \emptyset$ **then** (*p *is prime* *)
   2.1 $x := 2p$;
   2.2 **while** $x \leq X$ **do** (**remove multiples of* p *)
   2.2.1 $T[x] := T[x] \cup \{p\}$;
   2.2.2 $x := x + p$.

Postsieve: find all $x \in [2..X]$ s.t. $T[x] = \emptyset$.

Rem. replace 2.2 by $x := p^2$; 2.3.2 by $x := x + 2p$ as soon as $p > 2$. Some tricks are available (Brent, etc.).

II. Linked lists

Many pictures: (1, 3, 2, 4) or
`-> 1 -> 3 -> 2 -> 4 -> null`.

Which type of object? we store pairs (**`int`**, arrow).
An arrow is the adress of the following pair.

$\Rightarrow$ a linked list is formed with links.

Recursive definition: linked list $= \emptyset$ or $(link, linkedlist)$.

Array or list?

We use an array when:

- one knows the size (or a close upper bound) in advance;
- one needs to access the different cells in a random order (direct access to **`t[i]`**).

We use a list when:

- one doesn't know the size *a priori*;
- one doesn't need to access the i-th element;
- one has to perform additions/suppressions of items in head or tail;
- one wants to save space.

Definitions in C

We mimic a module with specification/implementation.

File `list.h`:

```
/* basic type */
typedef struct __list {
    int c;
    struct __list *next;
} __list, *list;

/* exported functions */
extern list list_alloc();
extern list_create(int);
extern list_add_first(int, list);
extern int list_length(list L);
```


First functions

File `list.c`:

```
#include <stdlib.h> /* standard library for malloc
#include "list.h" /* we use our list module */

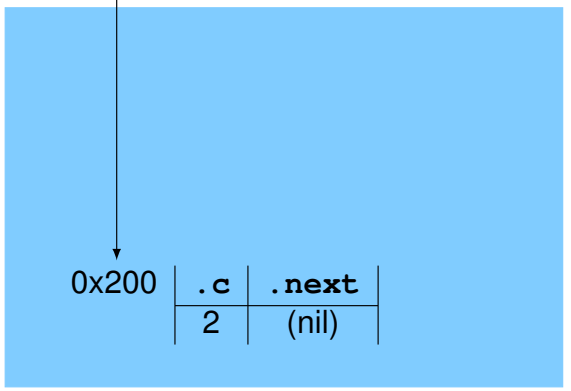
/* OUTPUT: -> */
list list_alloc(){
    return (list)malloc(sizeof(__list));
}
/* OUTPUT: -> c -> NULL */
list list_create(int c){
    list tmp = list_alloc();
    tmp->c = c;
    tmp->next = NULL;
    return tmp;
}
/* returns c -> L; cost O(1) */
list list_add_first(int c, list L){
    list tmp = list_alloc();
    tmp->c = c;
    tmp->next = L;
    return tmp;
}
```

What happens in memory

```
link L = NULL; //(nil)
```


What happens in memory

```
link L = NULL; //(nil)
L = list_add_first(2, L); //0x200
```



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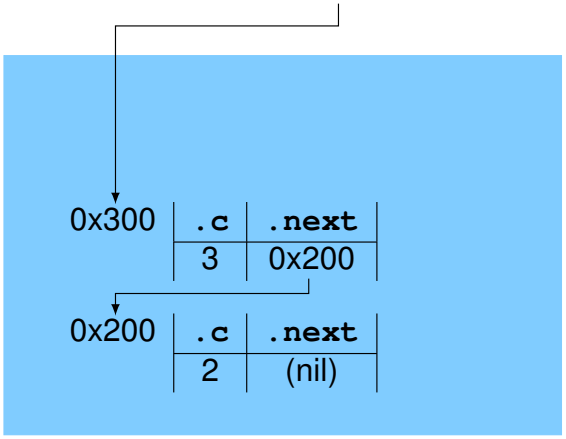
```



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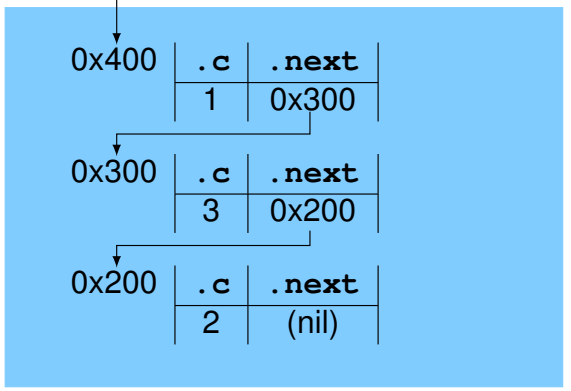
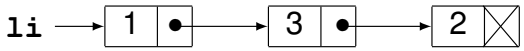
L = list_add_first(2, L); //0x200
L = list_add_first(3, L); //0x300
L = list_add_first(1, L); //0x400
```



Other pictures

Any of:
li = (1, 3, 2)
or `li -> 1 -> 3 -> 2 -> null`

More abstract:



Closer to memory:

```
li -> 0x400:[1, 0x300] -> 0x300:[3, 0x200]
      -> 0x200:[2, (nil)] -> (nil)
```


Two basic functions

```
/* OUTPUT: length of the list */
int list_length(list L){
    if(L == NULL)
        return 0;
    else
        return 1 + list_length(L->next);
}

/* OUTPUT: if c is in L, return the first part of
list list_is_in(int c, list L){
    if(L == NULL || L->c == c)
        return L;
    return list_is_in(c, L->next);
}
```


Removing and freeing

```
void list_free(list L){
    if(L != NULL){
        list_free(L->next);
        free(L);
    }
}
/* [c, L] outputs L */
list list_remove_first(int *ptr_c, list L){
    list tmp;

    if(L == NULL){
        *ptr_c = -1; /* why not? */
        return L;
    }
    *ptr_c = L->c;
    tmp    = L->next;
    free(L);
    return tmp;
}
```


Complexity issues

Adding/deleting head element: $O(1)$.
Adding/deleting tail element: $O(n)$ if L has size n.

To do better: doubly linked list

```
L -> 1 <-> 3 <-> 2 <-> 4 -> NULL
```

Need modify the structure:

```
typedef struct __dlist {
    int c;
    struct __dlist *next, *prev;
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```

Even better: double ended queue (dequeue) with a pointer on the head as well as on the tail.

Ex. write allocation, insertion/deletion head or tail that cost $O(1)$ operations.

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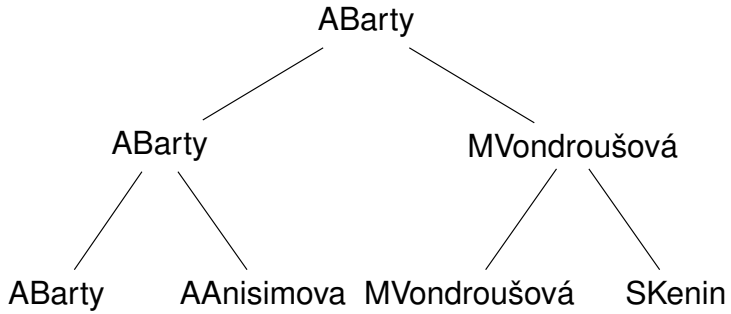
III. Trees

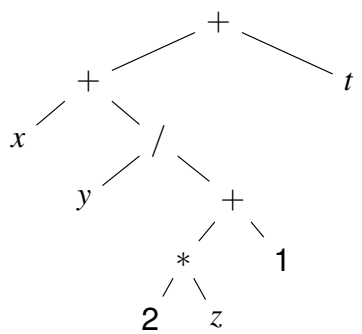
A) Introduction

Roland-Garros 2019:



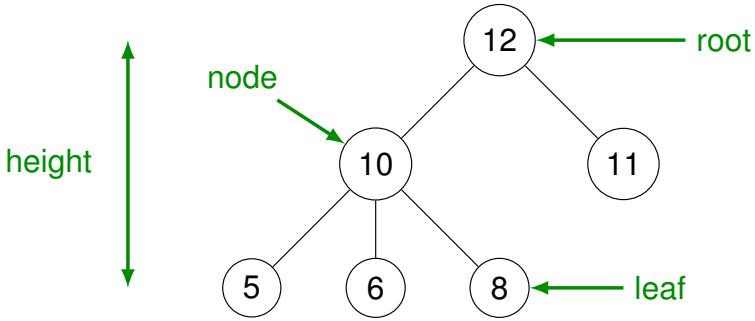
Another example

Representing $x+y/(2z+1)+t$:

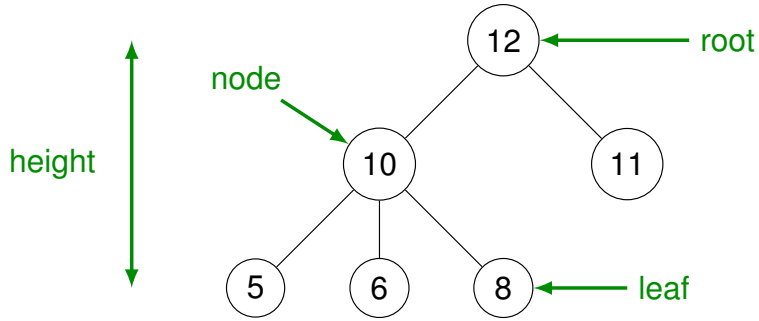


Useful also in compilation.

Terminology

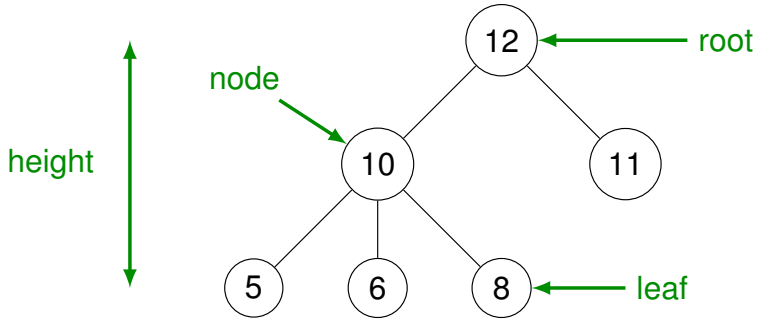


Terminology



The tree has 6 nodes: one root (12), 1 internal node (10), 4 leaves (5, 6, 8, 11); 10 is a child of 12; the tree has height 3 (the empty tree has height 0).

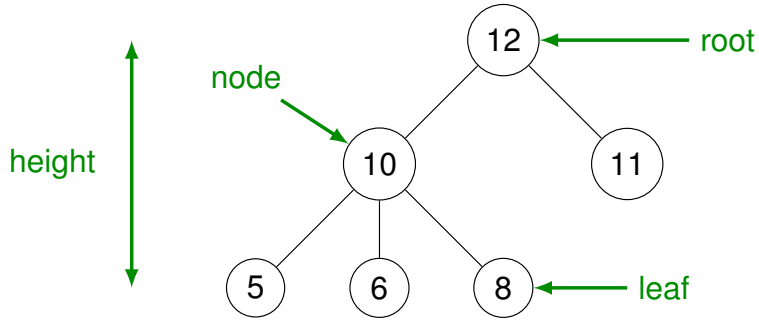
Terminology



The tree has 6 nodes: one root (12), 1 internal node (10), 4 leaves (5, 6, 8, 11); 10 is a child of 12; the tree has height 3 (the empty tree has height 0).

All nodes except the root have a parent, the leaves have no children.

Terminology



The tree has 6 nodes: one root (12), 1 internal node (10), 4 leaves (5, 6, 8, 11); 10 is a child of 12; the tree has height 3 (the empty tree has height 0).

All nodes except the root have a parent, the leaves have no children.

(12, 10, 6) or (12, 11) are paths in the tree.

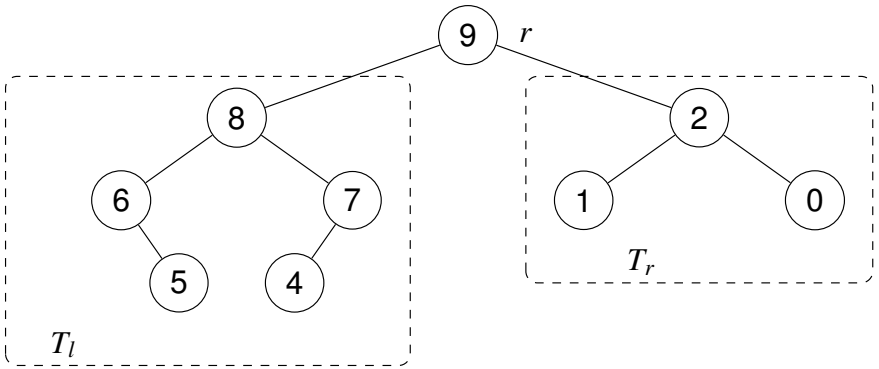
Required operations

- addition (insertion), suppression;
- searching for a given node;
- printing all nodes in hierarchical order (by level);
- following a branch (e.g., list of parents).

Rem. pictures, properties may not have any relationship with a given implementation.

B) Binary trees

Def. (recursive) $T = \emptyset$ or (r, T_l, T_r).



Dynamic implementation

Close to the recursive definition: $T = \emptyset$ ou (r, T_l, T_r).

File `tree.h`:

```
typedef struct node {
    int c;
    struct node *left, *right;
} node, *tree;
extern tree tree_alloc();
extern tree tree_create_leaf(int c);
```

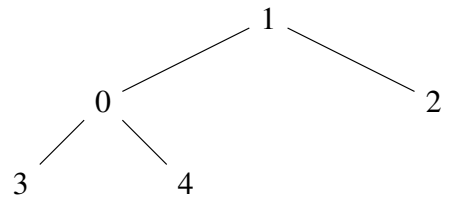
File `tree.c`:

```
tree tree_alloc(){
    return (tree)malloc(sizeof(node));
}
tree tree_create_leaf(int c){
    tree T = tree_alloc();
    T->left = NULL; T->right = NULL; T->c = c;
    return T;
}
```


Tests

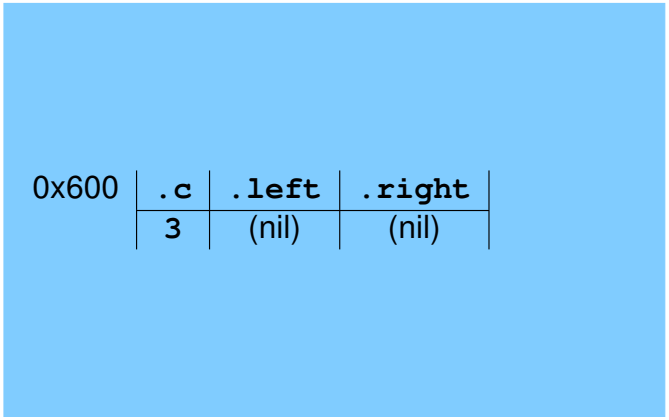
```
int main(int argc, char *argv[]){
    tree T = NULL, L, R, two;

    L = tree_create_leaf(3);
    R = tree_create_leaf(4);
    T = tree_join(0, L, R);
    two = tree_create_leaf(2);
    T = tree_join(1, T, two);
    return 0;
}
```



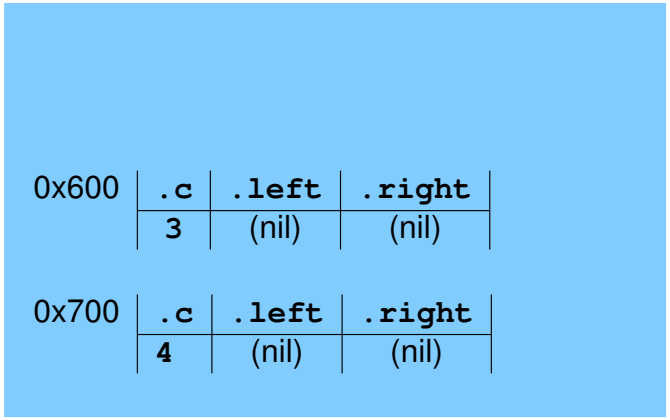
Memory picture

```
L = tree_create_leaf(3); //0x600
```



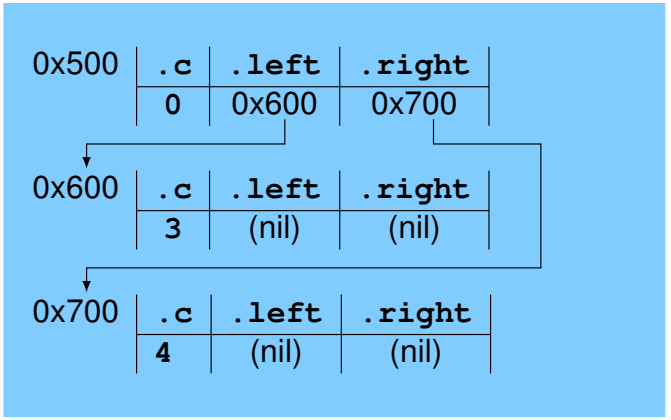
Memory picture

```
L = tree_create_leaf(3); //0x600
R = tree_create_leaf(4); //0x700
```



Memory picture

```
L = tree_create_leaf(3); //0x600
R = tree_create_leaf(4); //0x700
T = tree_join(0, L, R); //0x500
```



Tree traversals

Idea: inspect all nodes once.

Applications: printing, searching for a given information, etc.

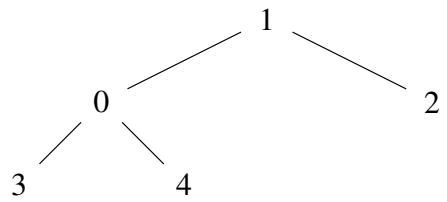
Classical traversals: on (r, T_l, T_r)

- prefix order: traverse r, then T_l, then T_r (closest to the definition);
- infix order: traverse T_l, then r, then T_r (boils down to flattening the tree);
- postfix order: traverse T_l, then T_r, then r.

The choice of the traversal depends on the application.

The three traversals for printing

```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
    if(T != NULL){
        printf(" %d", T->c);
        tree_prefix_print(T->left);
        tree_prefix_print(T->right);
    }
}
```



The three traversals for printing

```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
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        printf(" %d", T->c);
        tree_prefix_print(T->left);
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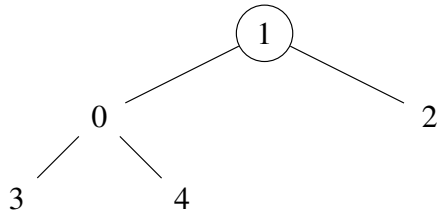
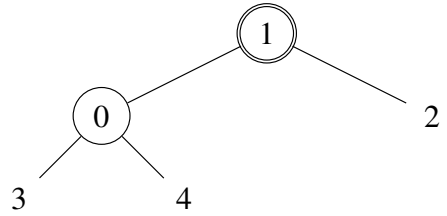
    }
}
```



$\Rightarrow 1$

The three traversals for printing

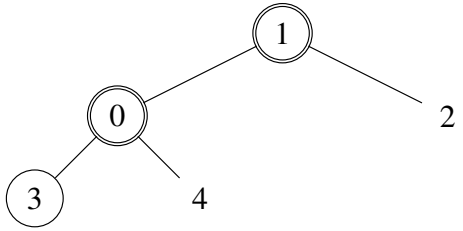
```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
    if(T != NULL){
        printf(" %d", T->c);
        tree_prefix_print(T->left);
        tree_prefix_print(T->right);
    }
}
```



$\Rightarrow 1,0$

The three traversals for printing

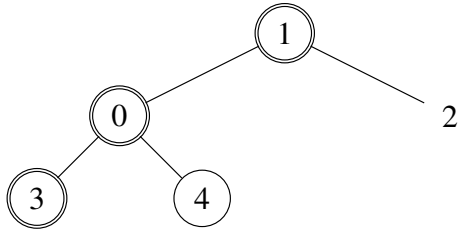
```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
    if(T != NULL){
        printf(" %d", T->c);
        tree_prefix_print(T->left);
        tree_prefix_print(T->right);
    }
}
```



$\Rightarrow 1,0,3$

The three traversals for printing

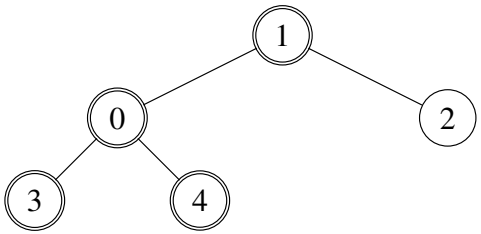
```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
    if(T != NULL){
        printf(" %d", T->c);
        tree_prefix_print(T->left);
        tree_prefix_print(T->right);
    }
}
```



$\Rightarrow 1,0,3,4$

The three traversals for printing

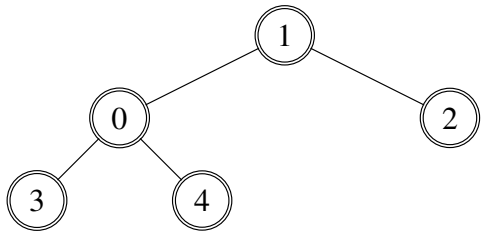
```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
    if(T != NULL){
        printf(" %d", T->c);
        tree_prefix_print(T->left);
        tree_prefix_print(T->right);
    }
}
```



$\Rightarrow 1,0,3,4,2$

The three traversals for printing

```
/* r, T_l, T_r */
void tree_prefix_print(tree T){
    if(T != NULL){
        printf(" %d", T->c);
        tree_prefix_print(T->left);
        tree_prefix_print(T->right);
    }
}
```



$\Rightarrow 1,0,3,4,2$

The other two traversals

```
// L, r, R
void tree_infix_print(tree T){
    if(T != NULL){
        tree_infix_print(T->left);
        printf(" %d", T->c);
        tree_infix_print(T->right);
    }
}
// L, R, r
void tree_postfix_print(tree T){
    if(T != NULL){
        tree_postfix_print(T->left);
        tree_postfix_print(T->right);
        printf(" %d", T->c);
    }
}
```


IV. Hashing

Pb: we want to store a set $\mathscr{E}$ having N integers (say) in a table, so that accessing any x costs as little as possible.

Ex. if $\mathscr{E} = [1,N]$, use `int t[N];` and `t[i] = -1` if empty.

We mimic this by computing for each $x \in \mathscr{E}$ an address $h(x) \in \{0,1,\ldots,M-1\}$ for some well chosen M. If $M \ll N$, several elements might fall at the same address (collision).

Insertion of x: $O(1)$

- compute $h(x)$;
- $\mathscr{T}_{h(x)} := \mathscr{T}_{h(x)} \cup \{x\}$.

Searching x: $O(1)$

- compute $h(x)$;
- return $x \in \mathscr{T}_{h(x)}$?

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Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11,59,32,44,301,26,199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$										

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i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11								

$11 = 1 \times 10 + 1 \Rightarrow 11 \bmod 10 = 1$

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11								59

$59 = 5 \times 10 + 9 \Rightarrow 59 \bmod 10 = 9$

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11	32							59

$32 = 3 \times 10 + 2 \Rightarrow 32 \bmod 10 = 2$

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11	32		44					59

$44 = 4 \times 10 + 4 \Rightarrow 44 \bmod 10 = 4$

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11, 301	32		44					59

$301 = 30 \times 10 + 1 \Rightarrow 301 \bmod 10 = 1$

collision!

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11, 301	32		44		26			59

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11, 301	32		44		26			59, 199

collision!

Example

Hash function: $h(z) = z \bmod M$.

Ex. $\mathscr{E} = \{11, 59, 32, 44, 301, 26, 199\}$; $M = 10$; $\mathscr{T}$ is an array of lists:

i	0	1	2	3	4	5	6	7	8	9
$\mathscr{T}_i$		11, 301	32		44		26			59, 199

Rem. We cannot avoid collisions, but if h is well chosen, the lists do not explode.

Rem. We can hash every object we want, since we can map any object to a string then to an integer.

Handling collisions using lists

```
#define HMAX 256
#define HMOD 256

/* fast way of computing x mod 2^n */
#define h(x) ((x) & (HMOD-1))

extern void hash_init(list H[]);
extern void hash_insert(list H[], int n);
extern int hash_is_in(list H[], int n);
extern void hash_print(list H[]);
```

hash.c

```
void hash_init(list H[]){
    int i;

    for(i = 0; i < HMOD; i++)
        H[i] = NULL;
}

void hash_insert(list H[], int n){
    int hn = h(n);

    if(list_is_in(n, H[hn]) == NULL)
        H[hn] = list_add_first(n, H[hn]);
}

int hash_is_in(list H[], int n){
    int hn = h(n);

    return list_is_in(n, H[hn]) != NULL;
}
```


hash.c

```
void hash_init(list H[]){
    int i;

    for(i = 0; i < HMOD; i++)
        H[i] = NULL;
}

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    return list_is_in(n, H[hn]) != NULL;
}
```


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void hash_init(list H[]){
    int i;

    for(i = 0; i < HMOD; i++)
        H[i] = NULL;
}

void hash_insert(list H[], int n){
    int hn = h(n);

    if(list_is_in(n, H[hn]) == NULL)
        H[hn] = list_add_first(n, H[hn]);
}

int hash_is_in(list H[], int n){
    int hn = h(n);

    return list_is_in(n, H[hn]) != NULL;
}
```


hash.c (cont'd)

```
void hash_print(list H[]){
    int i;
    for(i = 0; i < HMOD; i++)
        if(H[i] != NULL){
            printf("H[%d]=", i);
            list_print(H[i]);
            printf("\n");
        }
}
```

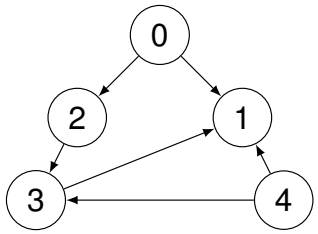
Example

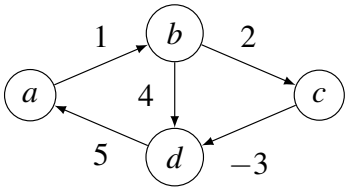
```
#include <stdio.h>
#include <stdlib.h>
#include "list.h"
#include "hash.h"

int main(int argc, char *argv[]){
    list tabh[HMAX];
    int i;
    hash_init(tabh);
    for(i = 0; i < 100; i++)
        hash_insert(tabh, abs(random()) % 1000);
    hash_print(tabh);
}
```

```
H[9]= -> 777
H[11]= -> 11
H[12]= -> 12
H[14]= -> 526 -> 782
...
```


V. Graphs





- Modelling networks: metro, train, electricity, INTERNET, etc.
- Representing logical links: ecosystems; dependency graph between source files, heritance diagram, etc.
- Ressource allocation: minimal cost, etc.

Representing graphs

Abstraction: $\mathscr{G} = (\mathscr{V}, \mathscr{E})$.

$\mathscr{V}$**:** set of vertices, e.g., $[0..n-1]$.

$\mathscr{E}$**:** set of edges, e.g., array of lists.

Ex.

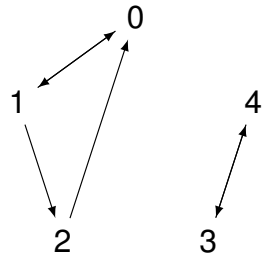
```
L[0] = (1, 2)
L[1] = ()
L[2] = (3)
L[3] = (2, 4)
L[4] = ()
```


When $\mathscr{V}$ is small: adjacency matrix

For a simple graph $\mathscr{G}$, matrix $n \times n$ s.t.

$$M_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \mathscr{E}, \\ 0 & \text{otherwise.} \end{cases}$$

Ex.



$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

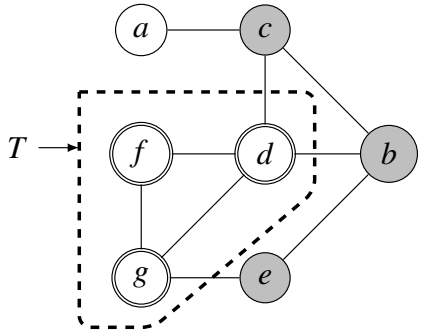
Graph with weights: $M_{i,j} = v(i,j)$.

Comparing both implementations

type	memory	utilisation
matrix	$O(n^2)$	dense graph
lists	$O(\|\mathscr{E}\|+n)$	sparse graph

Traversals (1/2)

Principle: starting from a vertex s, we traverse a subset of $\mathscr{G}$, the neighbours of s. Incrementally, we choose the next vertex in the frontier of the graph.



Def. The frontier $\mathscr{F}(T)$ of $T \subset \mathscr{V}$ is the set of vertices of $\mathscr{V} - T$ that are adjacent to one of the vertices of T.

Traversals (2/2)

Def. A traversal of $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ (connex) starting from s is a list of vertices L s.t.

- the first vertex of L is s;
- each vertex in $\mathscr{V}$ appears only once in L;
- each verext in L (except s) is adjacent in $\mathscr{G}$ to at least a vertex preceding in L.

Rem. There is no canonical (nor best) traversal.

Rem. The two most frequently used traversals are depth search first (DFS), and breadth first search (BFS).

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Rem. The two most frequently used traversals are depth search first (DFS), and breadth first search (BFS).

IV. DFS

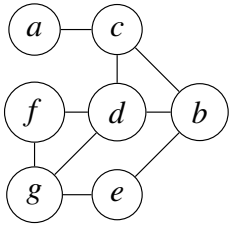
(*Depth First Search*) the next vertex to be traversed is the most recent vertex of type **beingexplored**.

```
function dfs(G, s)
0. forall vertex t do
       state[t] <- unexplored;
1. dfsRec(G, s, state);

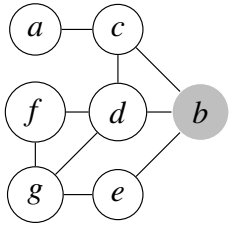
function dfsRec(G, s, state)
if state[s] == unexplored then
    state[s] <- beingexplored;
    forall t adjacent to s
        dfsRec(G, t, state);
    state[s] <- explored;
```

Thm. if all list operations cost $O(1)$, then the complexity of DFS is $O(|\mathscr{V}|+|\mathscr{E}|) = O(n+m)$.

Example



Example



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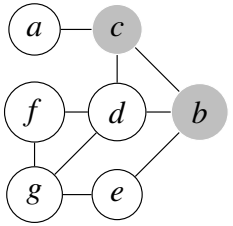

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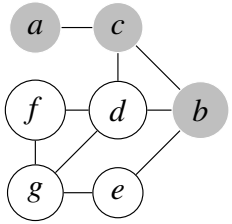

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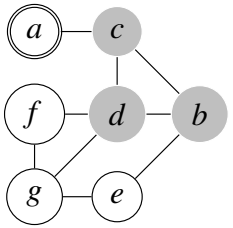

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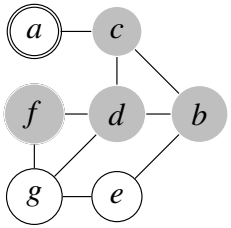

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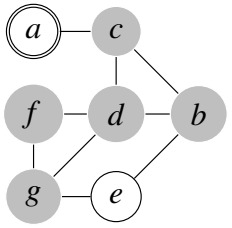

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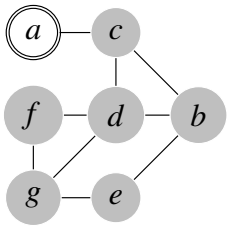

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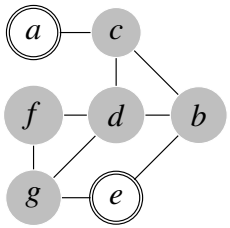

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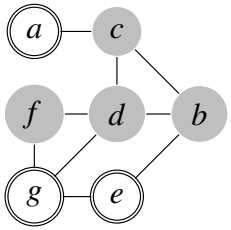

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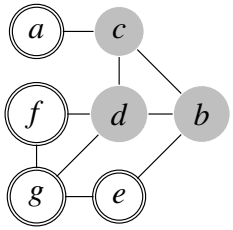

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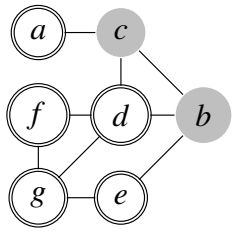

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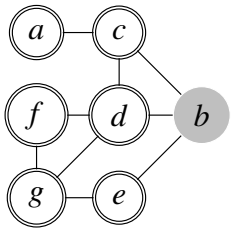

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