Probabilistic uncertainties and analysis of quantum programs:

Quantum computing is currently a hot topic, with massive investments in research from governments and private industries, as it holds promises of breaking complexity barriers, with major consequences for practical applications (AI, cryptography etc.). Before such a technology can be fully deployed, it is of primary importance to fully understand its underlying computational model, and find methods to automatically verify the safety of quantum programs.

Several methods have been proposed in the literature, some based on a logical view of quantum programs [CBD+22,BMLS22], others based on abstract interpretation [CC77,P21] which propagates outer-approximations of sets of quantum states throughout the program. This is the latter view that we are going to take in this internship. Quantum states will be abstracted first by their density operators [NC00], that is, a representation of probabilities to observe certain states. A first idea is to use the framework of PBoxes and Dempster-Shafer structures [AAOB+13], that are particular encodings of sets of probability distributions, to abstract these density operators. With this in mind, the objective is to define a calculus of “abstract” density operators that will represent tightly and efficiently a set of density operators, and will allow for computing the “abstract” effect of basic quantum constructs, such as the classical quantum gates [NC00].

To make this abstraction tight enough for practical purposes, we will need to encode some form of dependencies between quantum states, such as entanglement. A classical idea coming from probability theory would be to use some form of copula (some form of probabilistic dependency), which have been defined in the quantum context in e.g. [ALAA19], or as couplings in e.g. [GBJH+19]. As there is no obvious means to compute such copulas, an important objective will be to find a good abstraction of such probabilistic dependencies, either by elaborating on probabilistic analyzes such as

[AAOB+13,SSYC+20] (used in a more classical context), or by using ideas coming from symmetry actions (subgroups of the Pauli group on n qubits) on density operators [H15,P21], or from quantum epistemic logics [ABSS22].

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