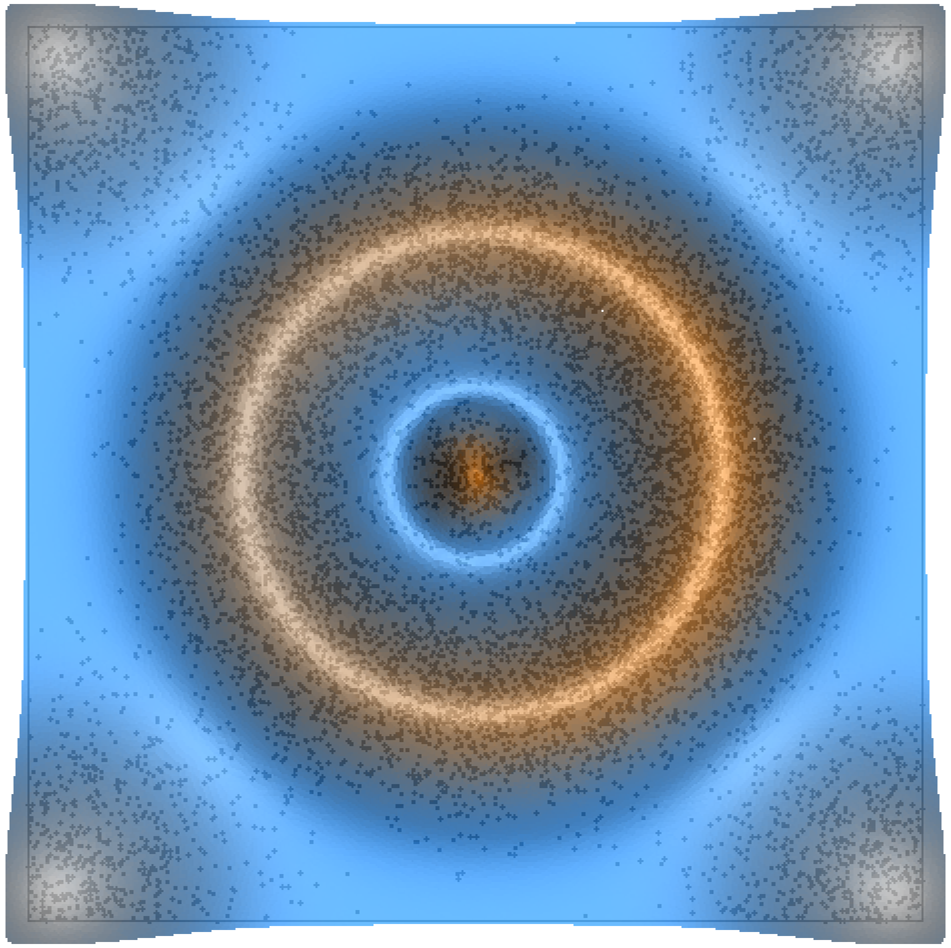
Mode-Seeking Paradigm

- Assume the data points are sampled from some unknown probability distribution
- Partition the data according to the basins of attraction of the peaks of the density



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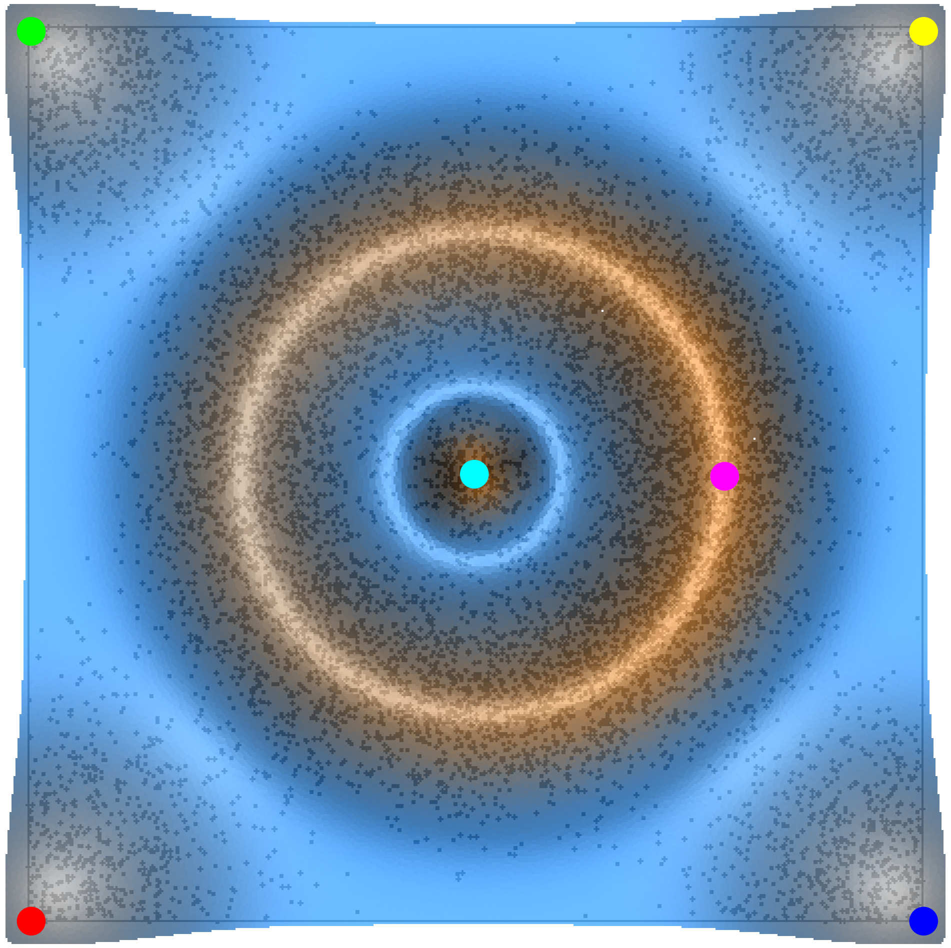

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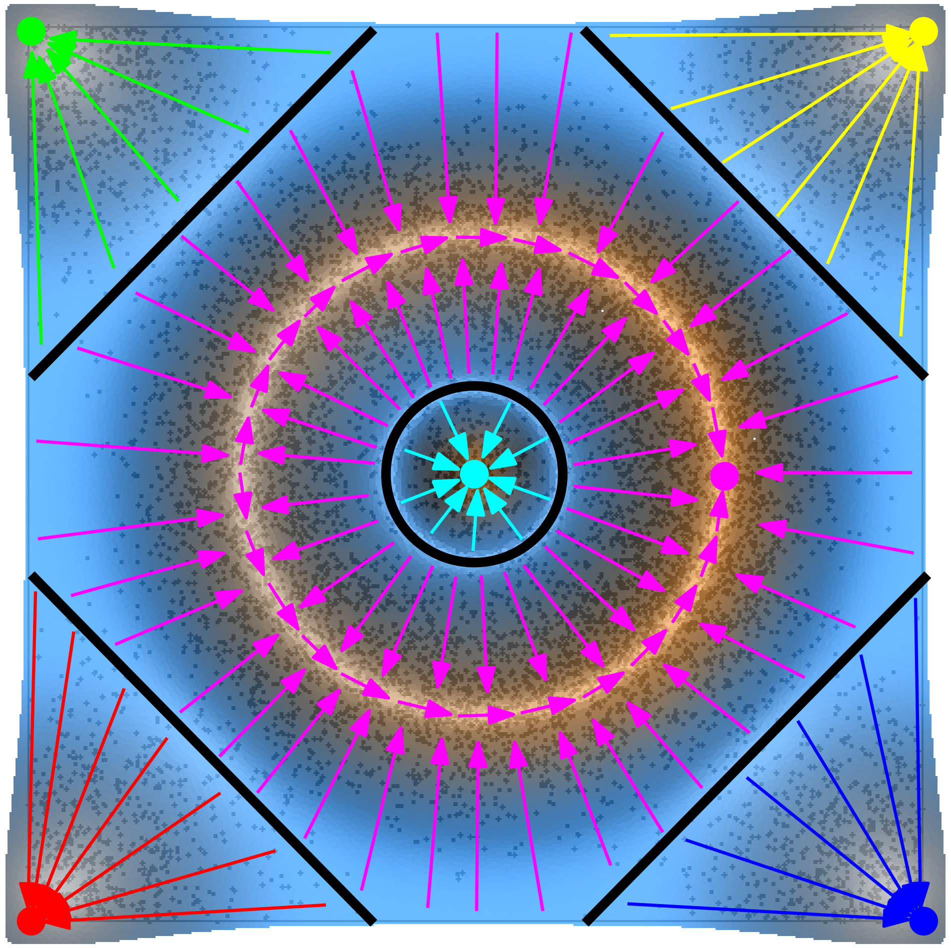

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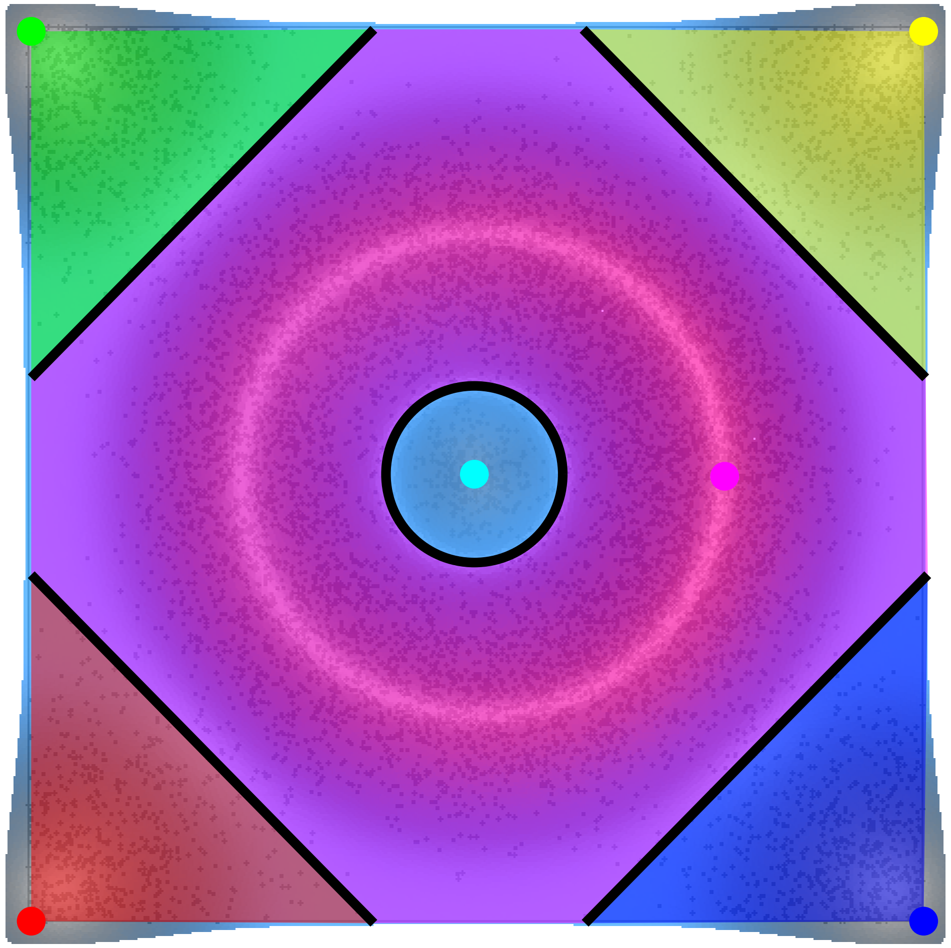

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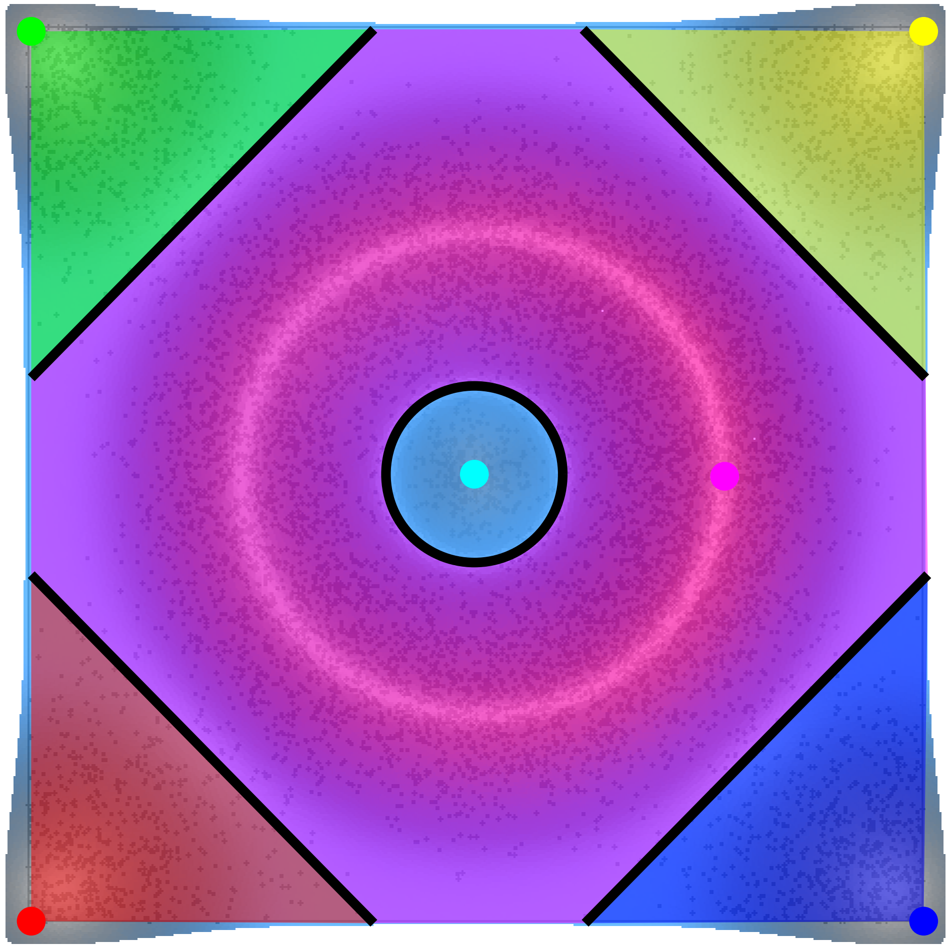
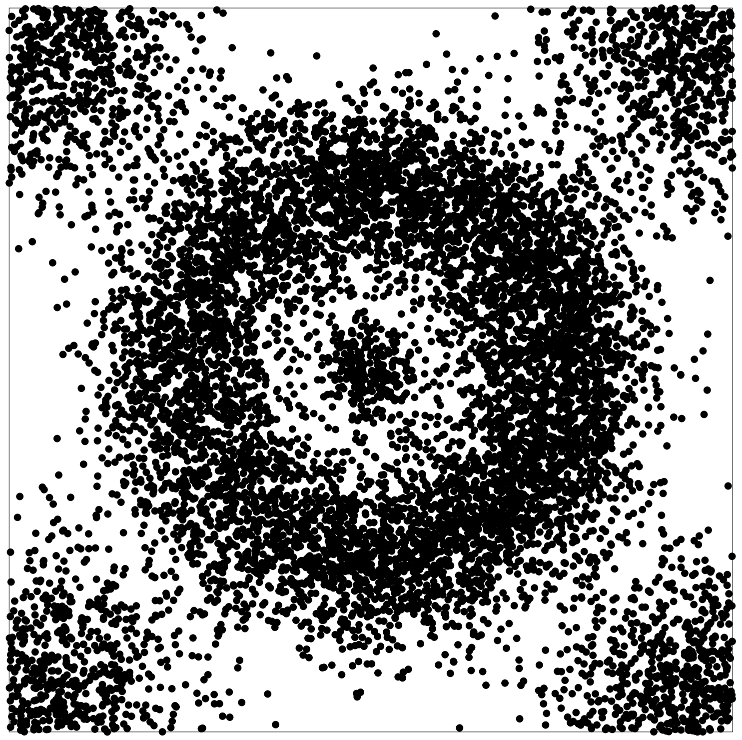


Hill-Climbing Schemes

- **Iterative**, e.g. D. Comaniciu and P. Meer. Mean shift: A robust approach toward feature space analysis. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 24(5):603619, May 2002.

- **Non-iterative**, e.g. W. L. Koontz, P. M. Narendra, and K. Fukunaga. A graph-theoretic approach to nonparametric cluster analysis. *IEEE Trans. on Computers*, 24:936944, September 1976.

[Koontz, Narendra, Fukunaga'76] in a Nutshell



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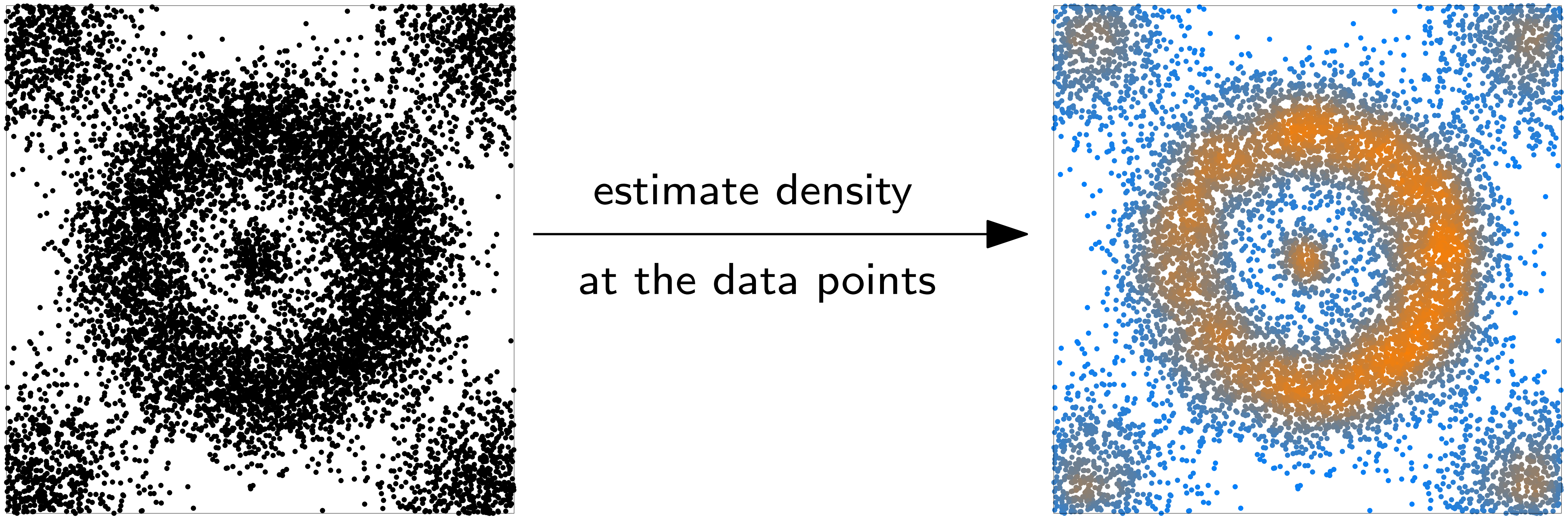

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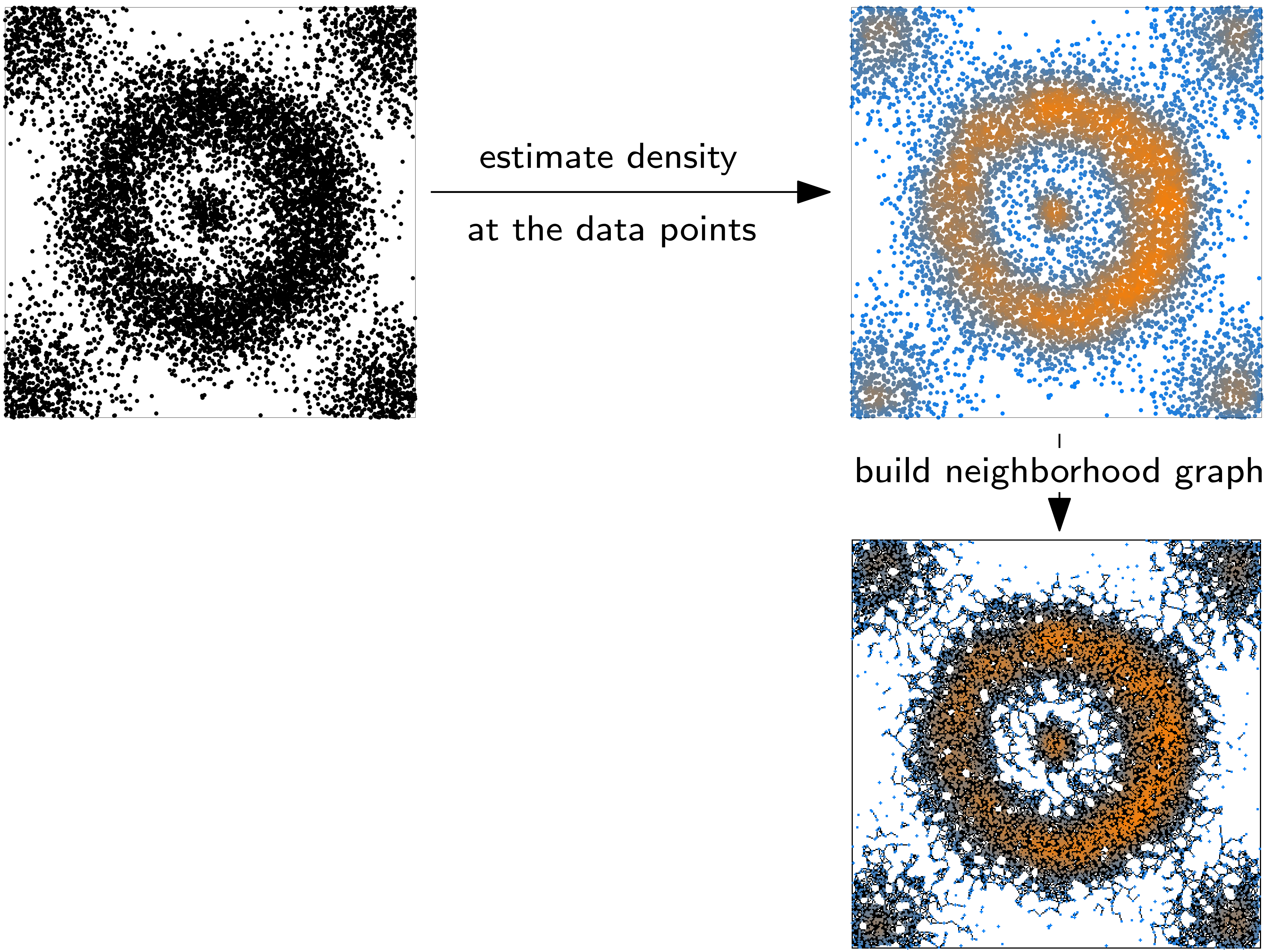

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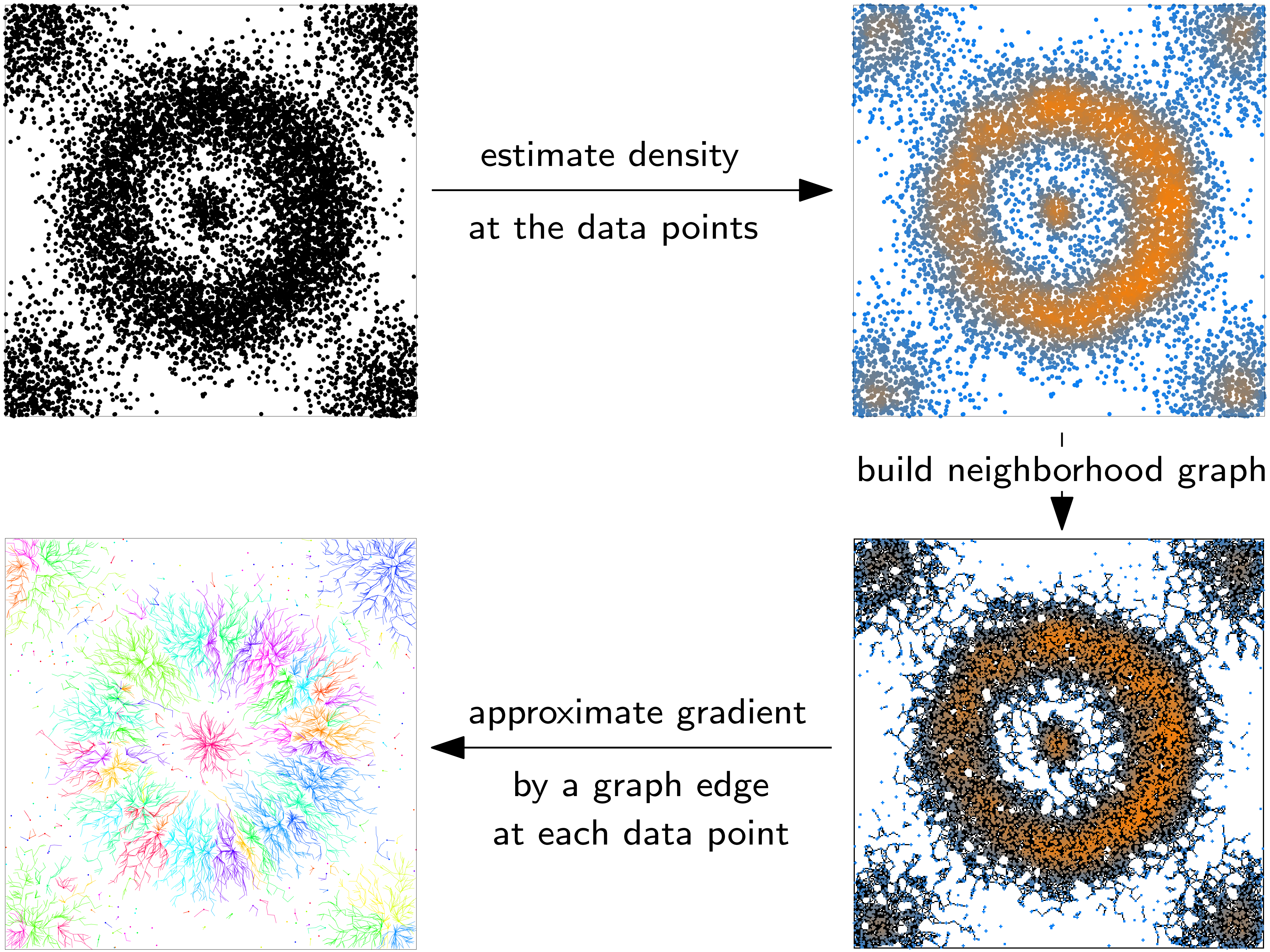
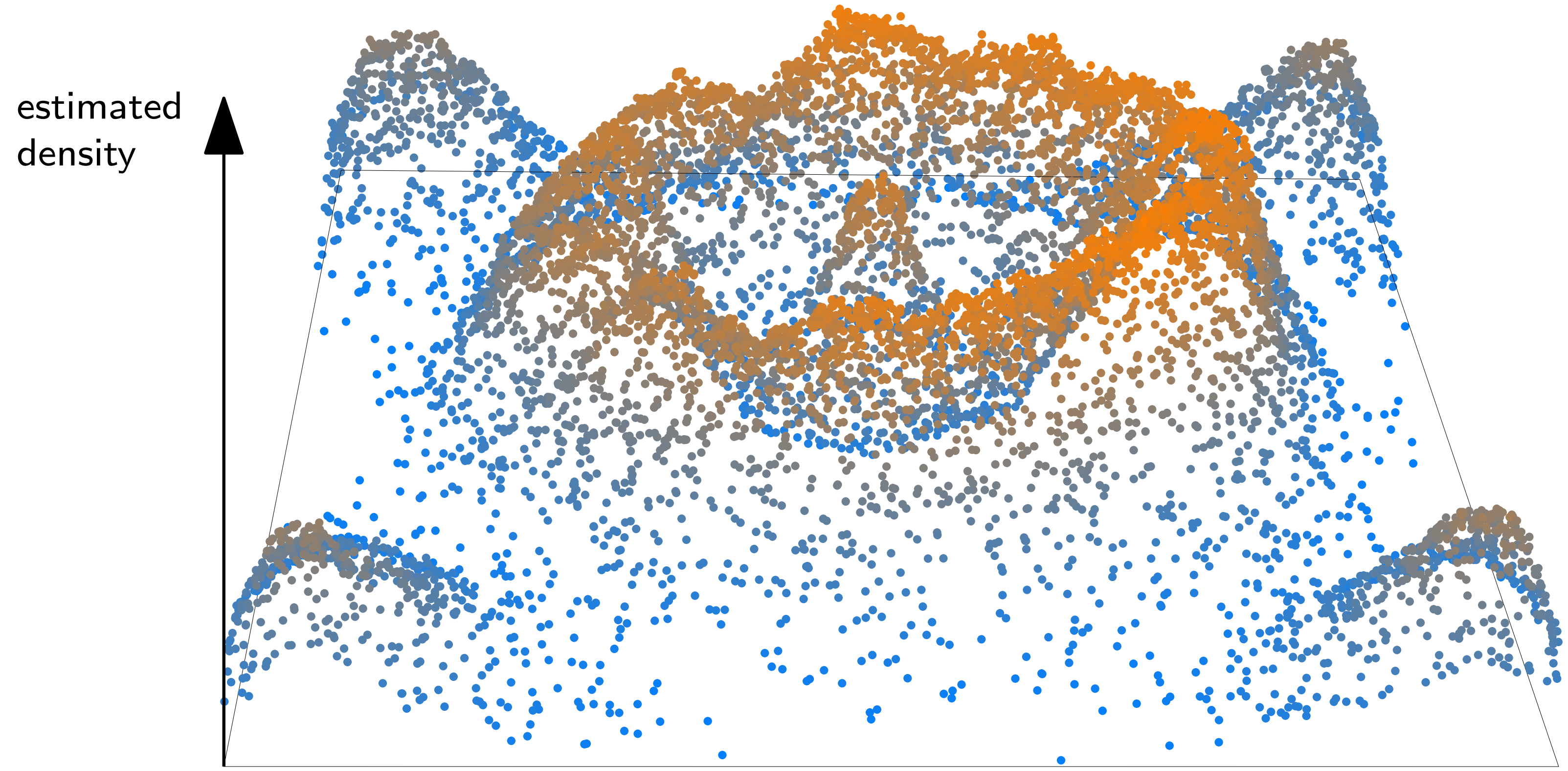
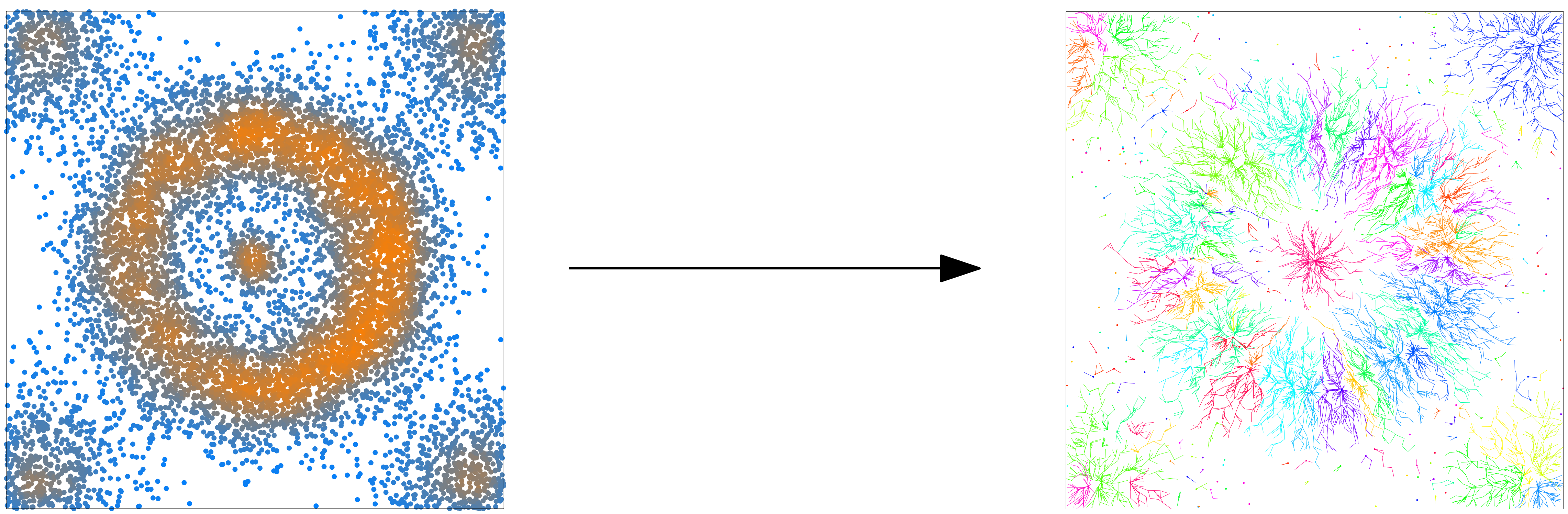


Why things are likely to go ill

- Noisy estimator





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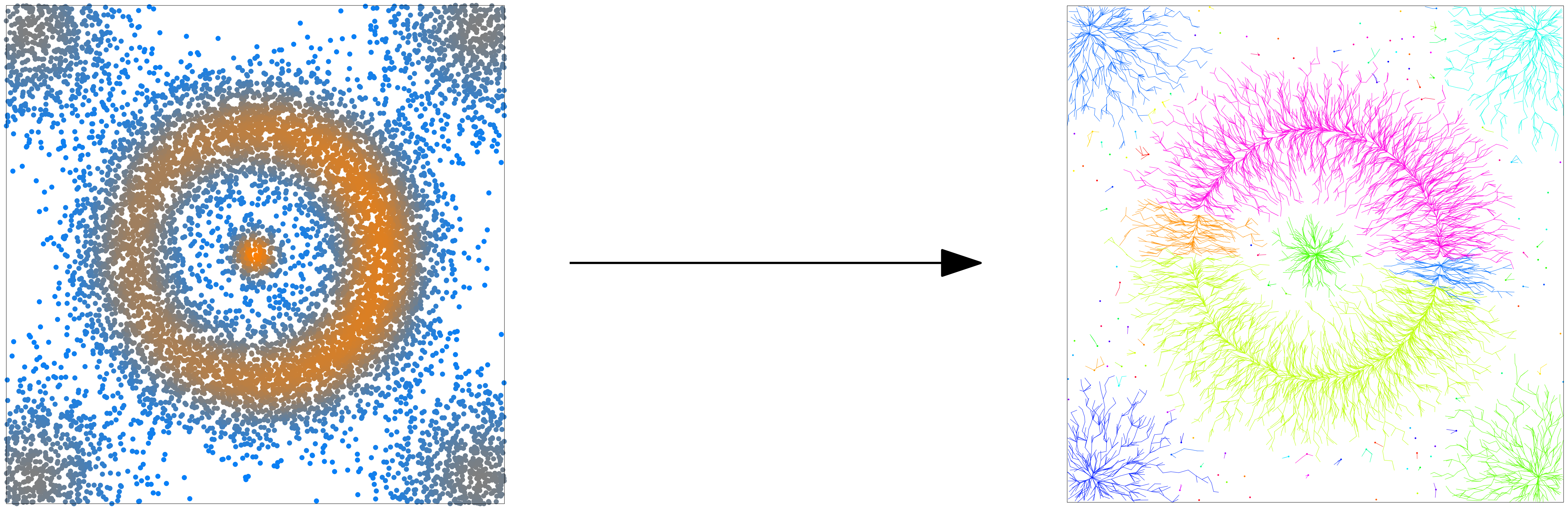

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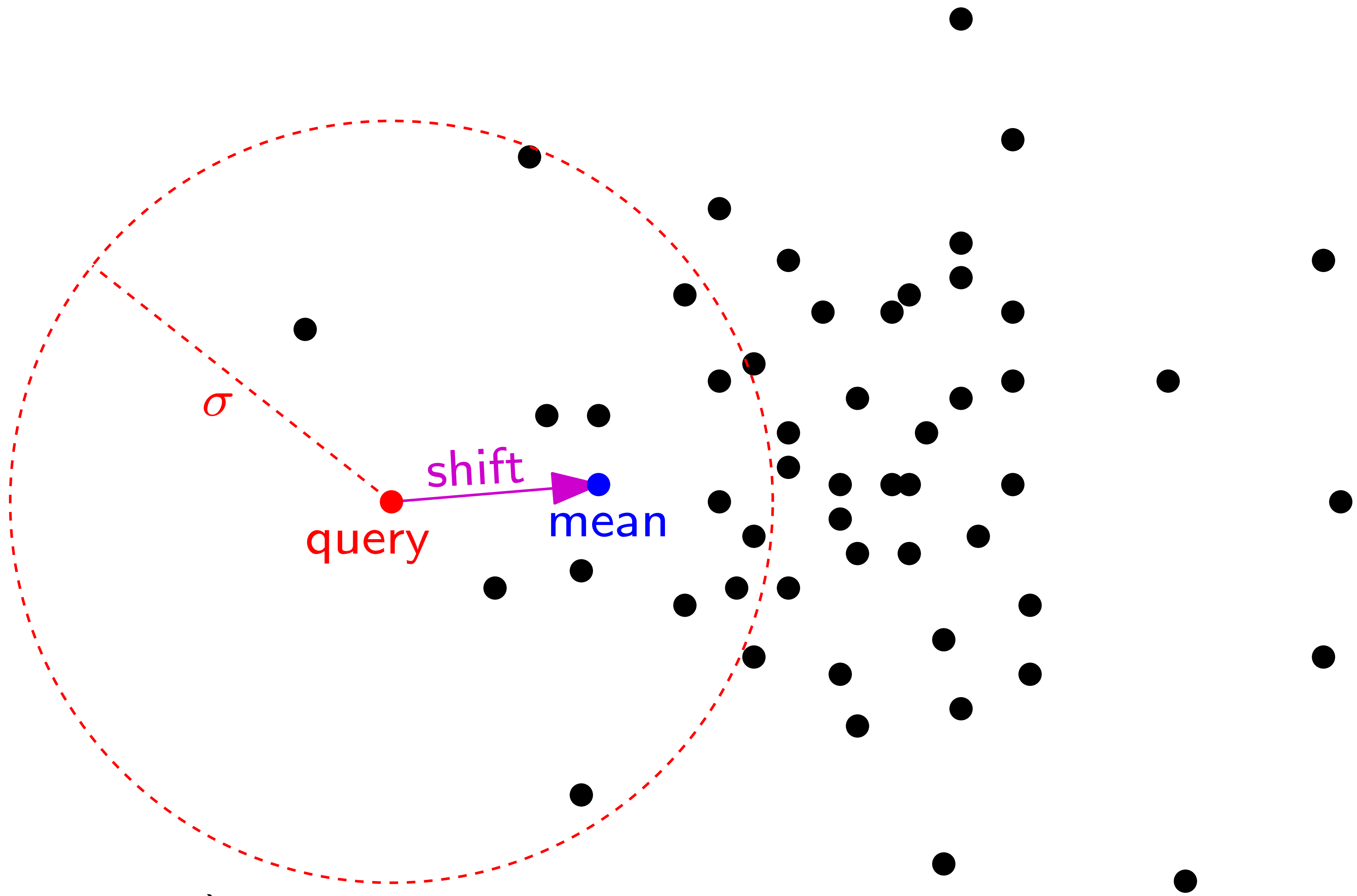
Mean-Shift in practice

- Apply Mean-Shift hill-climbing to each input point $p_i \in P$



(Epanechnikov kernel)

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- Gaussian kernel $\Rightarrow$ convergence at the limit (infinite time)

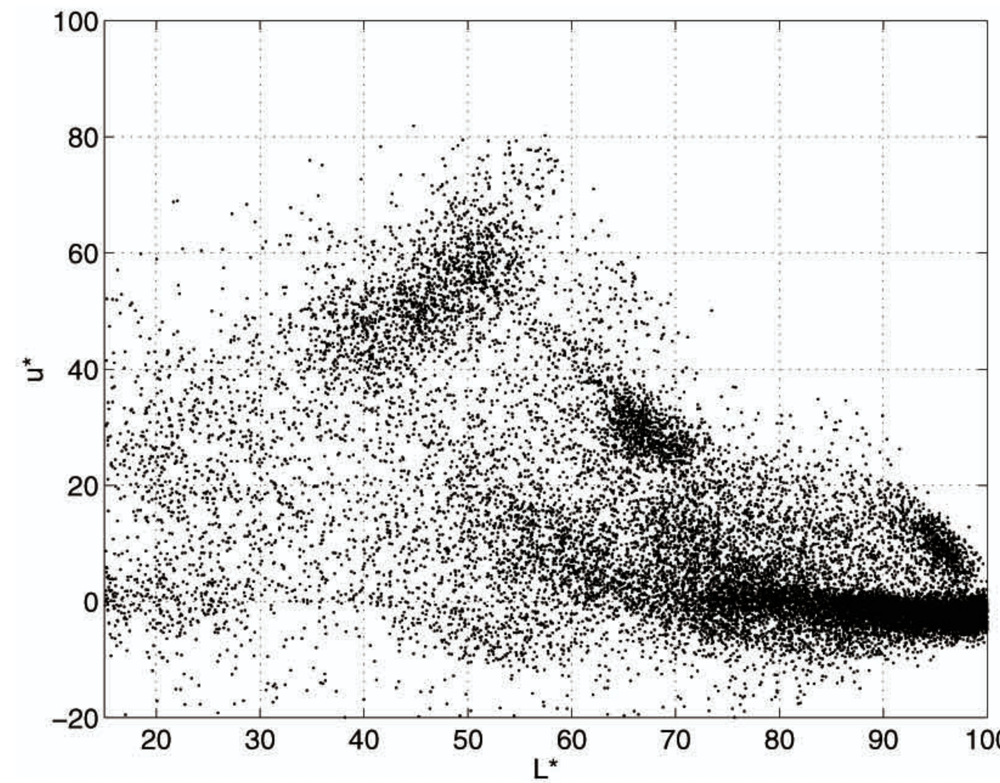
  $\rightarrow$ stopping criterion (convergence radius)

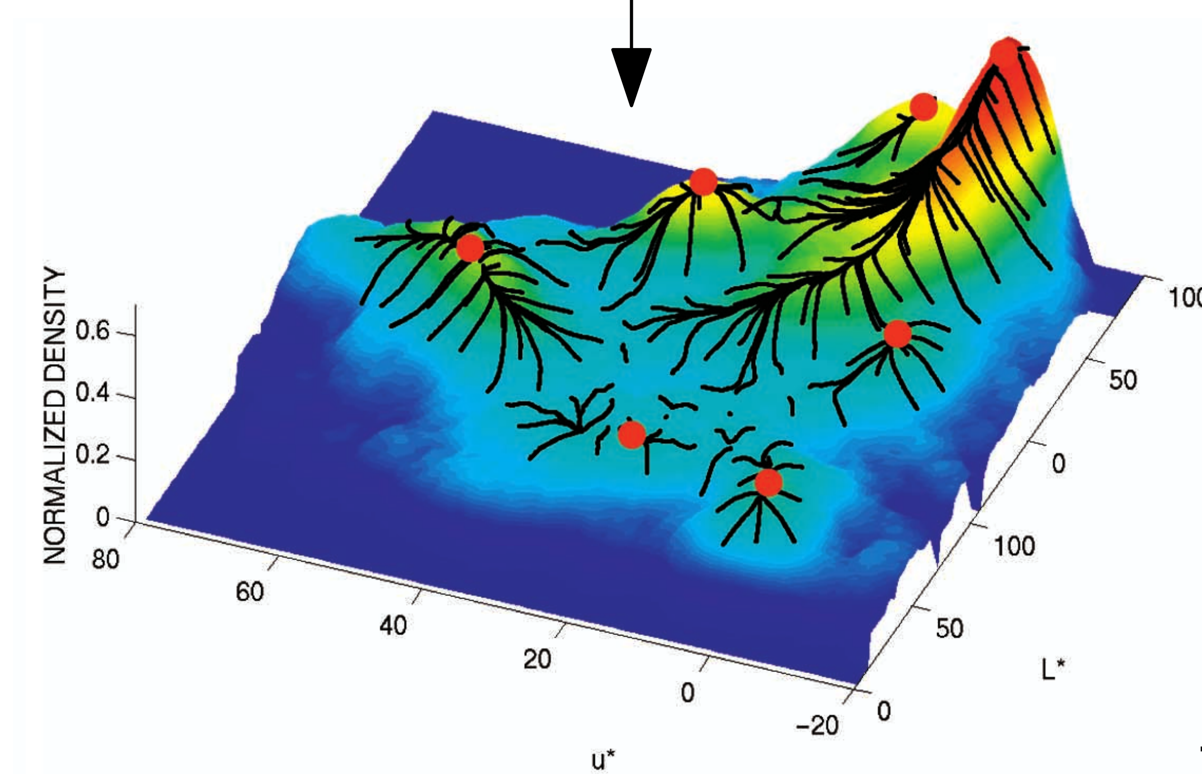
  $\rightarrow$ identification of modes (mode radius)

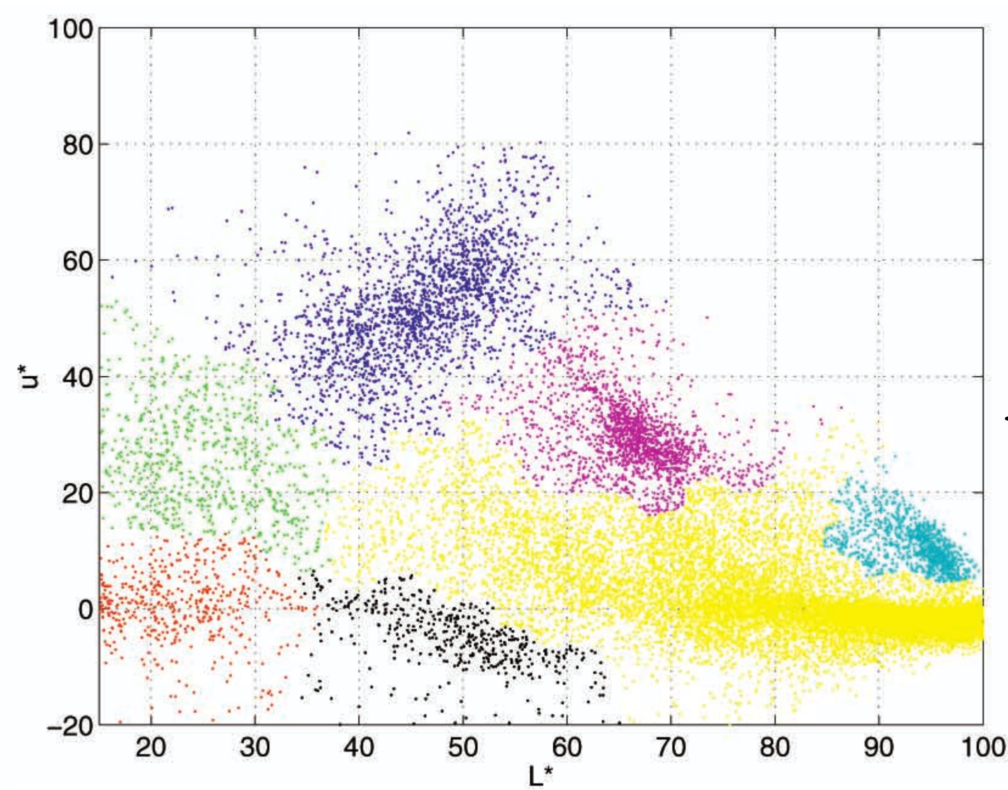
  $\rightarrow$ speed-up: hill-climbing gathers neighboring points (gathering radius)

$\rightsquigarrow$ heuristic: make these radii proportional to the estimator's bandwidth σ

Examples [Comaniciu, Meer 2002]







Examples [Comaniciu, Meer 2002]

Original

$(h_s, h_r) = (8, 8)$

$(h_s, h_r) = (8, 16)$

$(h_s, h_r) = (16, 4)$

$(h_s, h_r) = (16, 8)$

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$(h_s, h_r) = (32, 4)$

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