École Polytechnique

INF564 – Compilation

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static typing

if we write

```
    "5" + 37
```

do we get

- a compile-time error? (OCaml, Rust, Go)
- a runtime error? (Python, Julia)
- the integer 42? (Visual Basic, PHP)
- the string "537"? (Java, Scala, Kotlin)
- a pointer? (C, C++)
- something else?

and what about

```
  37 / "5"
```

?

if we now add two arbitrary expressions

```
    e1 + e2
```

how can we decide whether this is legal and which operation to perform?

the answer is **typing**, a program analysis that binds **types** to each sub-expression, to rule out inconsistent programs some languages are **dynamically typed**: types are bound to **values** and are used **at runtime**

examples: Lisp, PHP, Python, Julia

other languages are **statically typed**: types are bound to **expressions** and are used **at compile time**

examples: C, C++, Java, OCaml, Rust, Go

a language may use **both** static and dynamic typing

we will illustrate it with Java at the end of this lecture

1. static typing
   1.1 type checking WHILE
   1.2 type safety
2. implementing type checking
3. subtyping
4. overloading

static typing

well-typed programs do not go wrong

- type checking must be **decidable**
- type checking must reject programs whose evaluation would fail; this is **type safety**
- type checking must not reject too many non-absurd programs; the type system must be **expressive**

1. any sub-expression is annotated with a type

   ```
   int f(int x) { int y = ((x:int)+(1:int):int); ... }
   ```

   type checking is easy but this is unmanageable for the programmer

2. only annotate variable declarations (C, C++, Java, etc.)

   ```
   int f(int x) { int y = x+1; return y; }
   ```

3. only annotate function parameters (C++ 11, Java 10)

   ```
   int f(int x) { var y = x+1; return y; }
   ```

4. no annotation at all ⇒ **type inference** (OCaml, Haskell, etc.)

   ```
   fun x -> x+1
   ```

let us consider the language WHILE from lecture 2

to make it more interesting, let us add **records**
(and any variable is a record)

e	$::=$		**expression**
	$\mid$	c	integer or Boolean constant
	$\mid$	x	variable
	$\mid$	$e.f$	field access
	$\mid$	$e\ op\ e$	binary operator $(+, <, \dots)$

s	$::=$		**statement**
	$\mid$	$e.f \leftarrow e$	assignment
	$\mid$	`if` e `then` s `else` s	conditional
	$\mid$	`while` e `do` s	loop
	$\mid$	$s;\ s$	sequence
	$\mid$	`skip`	do nothing

$x.a \leftarrow 0;$
$x.b \leftarrow 1;$
$\texttt{while}\ x.b < 100\ \texttt{do}$
$\quad x.b \leftarrow x.a + x.b;$
$\quad x.a \leftarrow x.b - x.a$

the notion of value from lecture 2 is updated

$$\begin{array}{rcll} v & ::= & & \textbf{value} \\ & | & n & \text{integer value} \\ & | & b & \text{Boolean value} \\ & | & x & \text{address (here the name of the variable)} \end{array}$$

we also update the environment E,
which now maps pairs (x, f) to values $E(x, f)$

we define a big-step operational semantics for expressions

$$E, e \twoheadrightarrow v$$

and a small-step operational semantics for statements

$$E, s \rightarrow E', s'$$

$$\frac{}{E, n \twoheadrightarrow n} \qquad \frac{}{E, b \twoheadrightarrow b}$$

$$\frac{}{E, x \twoheadrightarrow x}$$

$$\frac{E, e \twoheadrightarrow x \quad (x, f) \in \mathrm{dom}(E)}{E, e.f \twoheadrightarrow E(x, f)}$$

$$\frac{E, e_1 \twoheadrightarrow n_1 \quad E, e_2 \twoheadrightarrow n_2 \quad n = n_1 + n_2}{E, e_1 + e_2 \twoheadrightarrow n} \qquad \text{etc.}$$

$$\frac{E, e_1 \twoheadrightarrow x \qquad E, e_2 \twoheadrightarrow v \quad (x, f) \in \mathrm{dom}(E)}{E, e_1.f \leftarrow e_2 \rightarrow E\{(x, f) \mapsto v\}, \mathtt{skip}}$$

$$\frac{}{E, \mathtt{skip}; s \rightarrow E, s} \qquad \frac{E, s_1 \rightarrow E_1, s_1'}{E, s_1; s_2 \rightarrow E_1, s_1'; s_2}$$

$$\frac{E, e \twoheadrightarrow \mathtt{true}}{E, \mathtt{if}\ e\ \mathtt{then}\ s_1\ \mathtt{else}\ s_2 \rightarrow E, s_1} \qquad \frac{E, e \twoheadrightarrow \mathtt{false}}{E, \mathtt{if}\ e\ \mathtt{then}\ s_1\ \mathtt{else}\ s_2 \rightarrow E, s_2}$$

$$\frac{E, e \twoheadrightarrow \mathtt{true}}{E, \mathtt{while}\ e\ \mathtt{do}\ s \rightarrow E, s; \mathtt{while}\ e\ \mathtt{do}\ s}$$

$$\frac{E, e \twoheadrightarrow \mathtt{false}}{E, \mathtt{while}\ e\ \mathtt{do}\ s \rightarrow E, \mathtt{skip}}$$

we introduce **types**, with the following abstract syntax

$$
\begin{array}{rcll}
\tau & ::= & & \textbf{type} \\
& | & \texttt{int} & \text{type of integer values} \\
& | & \texttt{bool} & \text{type of Boolean values} \\
& | & \{f : \tau; \dots; f : \tau\} & \text{record type}
\end{array}
$$

the type of a variable is given by a **typing environment** Γ
(a function from variables to types)

the **typing judgment** is written

$$\Gamma \vdash e : \tau$$

and reads "in typing environment Γ, expression e has type τ"

we use inference rules to define $\Gamma \vdash e : \tau$

$$\frac{}{\Gamma \vdash n : \texttt{int}} \qquad \frac{}{\Gamma \vdash b : \texttt{bool}}$$

$$\frac{x \in \mathrm{dom}(\Gamma)}{\Gamma \vdash x : \Gamma(x)}$$

$$\frac{\Gamma \vdash e : \{\dots; f : \tau; \dots\}}{\Gamma \vdash e.f : \tau}$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}}$$

etc.

with $\Gamma = \{x \mapsto \{a : \texttt{int}; b : \texttt{int}\}\}$, we have

$$\dfrac{\dfrac{\overline{\Gamma \vdash x : \{a : \texttt{int}; b : \texttt{int}\}}}{\Gamma \vdash x.a : \texttt{int}} \qquad \overline{\Gamma \vdash 1 : \texttt{int}}}{\Gamma \vdash x.a + 1 : \texttt{int}}$$

this derivation is a proof that `x.a+1` is well-typed in the same environment, we cannot type expressions such as

$$x.c$$

or

$$42.a$$

or

$$1 + \texttt{true}$$

this is precisely what we want, for these expressions have no value in our semantics to type statements, we introduce a new judgment

$$\Gamma \vdash s$$

that reads "in environment Γ, statement s is well-typed"

$$\frac{}{\Gamma \vdash \texttt{skip}} \qquad \frac{\Gamma \vdash s_1 \quad \Gamma \vdash s_2}{\Gamma \vdash s_1; s_2}$$

$$\frac{\Gamma \vdash e_1 : \{\dots; f : \tau : \dots\} \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1.f \leftarrow e_2}$$

$$\frac{\Gamma \vdash e : \texttt{bool} \qquad \Gamma \vdash s_1 \qquad \Gamma \vdash s_2}{\Gamma \vdash \texttt{if}\ e\ \texttt{then}\ s_1\ \texttt{else}\ s_2}$$

$$\frac{\Gamma \vdash e : \texttt{bool} \qquad \Gamma \vdash s}{\Gamma \vdash \texttt{while}\ e\ \texttt{do}\ s}$$

well-typed programs do not go wrong

let us show that our type system is safe wrt our operational semantics

Theorem (type safety)

If $\Gamma \vdash s$, then the reduction s is either infinite or reaches `skip`.

or, equivalently,

Theorem

If $\Gamma \vdash s$ and $E, s \rightarrow^\star E', s'$ and s' is irreducible, then s' is `skip`.

this means evaluation won't be **stuck** or any expression such as

$$42.a$$

or on a statement

$$\texttt{if } e \texttt{ then } s_1 \texttt{ else } s_2$$

where e does not evaluate to either `true` or `false` let us show first that well-typed expressions do evaluate successfully

$$\text{if } \Gamma \vdash e : \tau, \text{ then } E, e \twoheadrightarrow v$$

stated as such, this is not correct: we need a relationship between Γ and E

counterexample:

$$\begin{array}{rcl}\Gamma & = & \{x \mapsto \{a : \texttt{int}\}\} \\ e & = & x.a \\ E & = & \emptyset\end{array}$$

Definition (well-typed environment)

An execution environment E is well-typed in a typing environment Γ, written $\Gamma \vdash E$, if

$$\forall x,\ \textit{if}\ \Gamma(x) = \{\dots f : \tau \dots\}\ \textit{then}\ (x, f) \in \textit{dom}(E)\ \textit{and}\ \Gamma \vdash E(x, f) : \tau.$$

Lemma (evaluation of a well-typed expression)

If $\Gamma \vdash e : \tau$ and $\Gamma \vdash E$, then $E, e \twoheadrightarrow v$ and $\Gamma \vdash v : \tau$.

proof: by induction on the derivation $\Gamma \vdash e : \tau$.

$e = c$ immediate with $v = c$

$e = x$ immediate with $v = x$

$e = e_1.f$ by IH $E, e_1 \twoheadrightarrow v_1$ and $\Gamma \vdash v_1 : \tau_1$ with $\tau_1 = \{\dots f : \tau \dots\}$.
so v_1 is a variable x and $v = E(x, f)$
since E is well-typed, we have $\Gamma \vdash v : \tau$

$e = e_1 + e_2$ by IH on e_1 and e_2 we have $E, e_i \twoheadrightarrow v_i$ and $\Gamma \vdash v_i : \mathtt{int}$,
so v_1 and v_2 are integers and we conclude with $v = v_1 + v_2$

$\square$

the type safety proof is based on two lemmas

Lemma (progress)

If $\Gamma \vdash s$ and $\Gamma \vdash E$, then either s is `skip`*, or $E, s \rightarrow E', s'$.*

Lemma (preservation)

If $\Gamma \vdash s$, if $\Gamma \vdash E$ and if $E, s \rightarrow E', s'$ then $\Gamma \vdash s'$ and $\Gamma \vdash E'$.

Lemma (progress)

If $\Gamma \vdash s$ and $\Gamma \vdash E$, then either s is `skip`*, or $E, s \rightarrow E', s'$.*

proof: by induction on the derivation $\Gamma \vdash s$

$s = \texttt{skip}$ immediate

$s = s_1; s_2$ if $s_1 = \texttt{skip}$, we have $E, s_1; s_2 \rightarrow E, s_2$
otherwise, we use IH on s_1, so $E, s_1 \rightarrow E', s_1'$ and thus
$E, s_1; s_2 \rightarrow E', s_1'; s_2$

$s = e_1.f \leftarrow e_2$ since e_1 and e_2 are well-typed, they evaluate to x and v respectively
since $\Gamma \vdash x : \{\dots f : \tau \dots\}$ we have $(x, f) \in \mathrm{dom}(E)$ and
thus $E, s \rightarrow E', \texttt{skip}$ with $E' = E\{(x, f) \mapsto v\}$

other cases left as exercise $\square$

then we show

Lemma (preservation)

If $\Gamma \vdash s$, if $\Gamma \vdash E$ and if $E, s \to E', s'$ then $\Gamma \vdash s'$ and $\Gamma \vdash E'$.

proof: by induction on the derivation $\Gamma \vdash s$

$s = s_1; s_2$ we have $\Gamma \vdash s_1$ and $\Gamma \vdash s_2$

- if $s_1 = \mathtt{skip}$, then $E, s_1; s_2 \to E, s_2$
- otherwise, $E, s_1 \to E', s_1'$ and by IH $\Gamma \vdash s_1'$ and $\Gamma \vdash E'$ so $\Gamma \vdash s_1'; s_2$

$s = e_1.f \leftarrow e_2$ we have $E, e_1 \twoheadrightarrow x$ and $E, e_2 \twoheadrightarrow v$ and $s' = \mathtt{skip}$ (so $\Gamma \vdash s'$) and $E' = E\{(x, f) \mapsto v\}$
but $\Gamma \vdash e_1 : \{\dots f : \tau \dots\}$ and $\Gamma \vdash e_2 : \tau$ so $\Gamma \vdash v : \tau$ (see slide **??**) and thus $\Gamma \vdash E'$

other cases left as exercise $\square$

now we can deduce type safety easily

Theorem (type safety)

If $\Gamma \vdash s$ and $E, s \rightarrow^\star E', s'$ and s' is irreducible, then s' is `skip`.

proof: we have $E, s \rightarrow E_1, s_1 \rightarrow \cdots \rightarrow E', s'$ and by repeated applications of the preservation lemma, we have $\Gamma \vdash s'$
by the progress lemma, s' is reducible or is `skip`
so this is `skip` $\square$

the lecture notes contains a similar proof for Mini-ML,
with types as follows

$$\begin{array}{rcll} \tau & ::= & \texttt{int} \mid \texttt{bool} \mid \dots & \textit{base types} \\ & \mid & \tau \rightarrow \tau & \textit{function type} \\ & \mid & \tau \times \tau & \textit{type of a pair} \end{array}$$

as we just did, the proof is based on progress and preservation properties

see chapter 5

languages such as Java or OCaml enjoy such a type safety property

which means that the evaluation of an expression of type τ

- either does not terminate
- or raises an exception
- or terminates on a value **with type** τ

in OCaml, the absence of `null` makes it a rather strong property

implementing type checking

there is a difference between **the typing rules**, which define the relation

$$\Gamma \vdash e : \tau$$

and the **type checking algorithm**, which checks that a given expression e is well-typed in some environment Γ

for instance

- the type τ is not necessarily given (type inference)
- several rules may apply for a single construct
- an expression may have several types

the case of WHILE is simple, as a single rule applies for each expression

we say that typing is **syntax-directed**

the type checking is then implemented with a linear time traversal of the program

- we do not simply say

```
type error
```

but we **explain** the type error precisely

- we **keep** types for the further phases of the compiler to do this, we **decorate** abstract syntax trees

- **input** of type checking contains positions in source code
- **output** of type checking contains types

$$\boxed{\begin{array}{c}\text{syntax trees}\\ \text{with locations}\end{array}} \xrightarrow{\text{type checker}} \boxed{\begin{array}{c}\text{syntax trees}\\ \text{with types}\end{array}}$$

in OCaml

```
type loc = ...
```

```
type expr =




| Evar   of string
| Econst of int
| Efield of expr * string
...
```

in Java

```
class Loc { ... }
```

```
abstract class Expr {



}
class Evar   extends Expr {...}
class Econst extends Expr {...}
class Efield extends Expr {...}
...
```

in OCaml

```
type loc = ...
```

```
type expr = {
  desc: desc;
  loc : loc;
}
and desc =
| Evar   of string
| Econst of int
| Efield of expr * string
...
```

in Java

```
class Loc { ... }
```

```
abstract class Expr {
  Loc loc;


}
class Evar   extends Expr {...}
class Econst extends Expr {...}
class Efield extends Expr {...}
...
```

we signal a type error with an exception

the exception contains

- a message explaining the error
- a position in the source code

we catch this exception in the main function

we display the position and the message

```
test.c:8:14: error: too few arguments to function 'f'
```

we set up an abstract syntax for types

```
type typ = ...
```

```
class Typ { ... }
```

and a **new** abstract syntax for programs

```
type texpr = {
  tdesc: tdesc;
  typ  : typ
}
and tdesc =
| Tvar   of string
| Tconst of int
| Tfield of texpr * string
...
```

```
abstract class Texpr {
  Typ typ;


}
class Tvar   extends Texpr {...}
class Tconst extends Texpr {...}
class Tfield extends Texpr {...}
...
```

the type checker turns a parsed syntax tree into **another**, typed syntax tree

$$\boxed{\text{parsed trees}} \xrightarrow{\text{type checker}} \boxed{\text{typed trees}}$$

yet this is efficient, since

- it is typically a linear traversal
- former AST are collected by the GC

subtyping

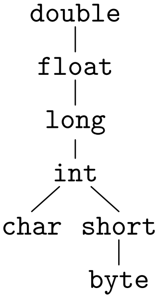
we say that a type τ_1 is a **subtype** of a type τ_2, which we write

$$\tau_1 \leq \tau_2$$

if any value with type τ_1 can be considered as a value with type τ_2

in many languages, there is subtyping between numerical types

in Java, it is as shown on the right



thus we can write

```
int n = 'a';
```

but not

```
byte b = 144;
```

in an object-oriented language, inheritance induces **subtyping**:
if a class `B` inherits from a class `A`, we have

$$\mathtt{B} \leq \mathtt{A}$$

i.e. any value of type `B` can be seen as a value of type `A` the two classes

```
class Vehicle                 { ... void move() { ... } ... }
class Car extends Vehicle { ... void move() { ... } ... }
```

induce the subtyping relation

$$\texttt{Car} \leq \texttt{Vehicle}$$

and thus we can write

```
Vehicle v = new Car();
v.move();
```

the construct `new C(...)` builds an object of class `C`, and the class of this object cannot be changed in the future; this is the **dynamic type** of the object

however, the **static type** of an expression, as computed by the compiler, may differ from the dynamic type, because of subtyping

when we write

```
Vehicle v = new Car();
v.move();
```

variable `v` has type `Vehicle`, but the method `move` that is called is that of class `Car` (we'll explain how in another lecture) in many cases, the compiler **cannot** determine the dynamic type

example:

```
void moveAll(LinkedList<Vehicule> l) {
  for (Vehicule v: l)
    v.move();
}
```

sometimes we need to force the compiler's hand, which means we claim that a value has some type

we call this **type casting** (or simply `cast`)

Java's notation, inherited from C, is

$$(\tau)e$$

the static type of this expression τ

using a cast, we can write

```
int  n = ...;
byte b = (byte)n;
```

in this case, there is no dynamic verification
(if the integer is too large, it is truncated)

let us consider

$$(C)e$$

where

- D is the dynamic type of (the object designated by) e
- E is the static type of expression e

there are three cases

- C is a super class of E: this is an **upcast** and the code for $(C)e$ is that of e (but the cast has some influence anyway, since $(C)e$ has type C)
- C is a subclass of E: this is a **downcast** and the code contains **dynamic test** to check that D is indeed a subclass of C
- C is neither a subclass nor a super of E: the compiler rejects the program with a type error

```
class A {
  int x = 1;
}

class B extends A {
  int x = 2;
}
```

```
  B b = new B();
  System.out.println(b.x);      // 2
  System.out.println(((A)b).x); // 1
```

```
void m(Vehicle v, Vehicle w) {
  ((Car)v).await(w);
}
```

nothing guarantees that the object passed to `m` will be a car; in particular, it could have no method `await`!

the dynamic test is required

Java raises `ClassCastException` is the test fails

testing subtyping dynamically

to allow defensive programming, there exists a Boolean construct

$$e \ \texttt{instanceof} \ C$$

that checks whether the class of e is indeed a subclass of C

it is idiomatic to do

```
if (e instanceof C) {
  C c = (C)e;
  ...
}
```

in this case, the compiler makes an optimization to perform a single test

overloading

(**surcharge** en français)

overloading is the ability to reuse the same name of several operations

overloading is handled **at compile time**, using the number and the (static) types of arguments

caveat: not to be confused with *overriding* (see lecture 6) in Java, operation + is overloaded

```
int    n = 40 + 2;
String s = "foo" + "bar";
String t = "foo" + 42;
```

these are three distinct operations

```
int    +(int   , int   )
String +(String, String)
String +(String, int   )
```

when we write

```
int n = 'a' + 42;
```

this is subtyping that allows us to consider `'a'` with type `char` as a value of type `int`, and thus the operation is `+(int, int)`

but when we write

```
String t = "foo" + 42;
```

this is **not** subtyping (`int` $\not\leq$ `String`)

in particular, we cannot write

```
String t = 42;
```

in Java, one cannot overload operators such as +
but one can overload methods/constructors

```
int f(int n, int m) { ... }
int f(int n)        { ... }
int f(String s)     { ... }
```

this is exactly as if we had written

```
int f_int_int(int n, int m) { ... }
int f_int    (int n)        { ... }
int f_String (String s)     { ... }
```

the compiler uses the static types of `f`'s arguments to determine which method to call yet overloading resolution can be tricky

```
class A {...}
class B extends A {
  void m(A a) {...}
  void m(B b) {...}
}
```

with

```
{ ... B b = new B(); b.m(b); ... }
```

both methods apply

this is method `m(B b)` that is called, because it is considered **more precise** some cases are ambiguous

```
class A {...}
class B extends A {
  void m(A a, B b) {...}
  void m(B b, A a) {...}
}
{ ... B b = new B(); b.m(b, b); ... }
```

and reported as such

```
test.java:13: reference to m is ambiguous,
  both method m(A,B) in B and method m(B,A) in B match
```

to each method defined in class C

$$\tau\ \mathtt{m}(\tau_1\ x_1, ..., \tau_n\ x_n)$$

we set the profile $(\mathtt{C}, \tau_1, \ldots, \tau_n)$

then we **order** profiles: $(\tau_0, \tau_1, \ldots, \tau_n) \sqsubseteq (\tau'_0, \tau'_1, \ldots, \tau'_n)$ if and only if τ_i is a subtype of τ'_i for all i

for a call

$$e.m(e_1, \ldots, e_n)$$

where e has static type τ_0 and e_i has static type τ_i, we consider the set of all **minimal** elements in the set of all compatible profiles

- no element $\Rightarrow$ no method applies
- several elements $\Rightarrow$ ambiguity
- a single element $\Rightarrow$ this is the method to call

next

- lab 4 = we start the project
  - static typing of Mini Python
  - (the parser is provided)

- next lecture
  - evaluation strategy
  - parameter passing