École Polytechnique

INF564 – Compilation

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static typing
if we write

"5" + 37

do we get

• a compile-time error? (OCaml, Rust, Go)
• a runtime error? (Python, Julia)
• the integer 42? (Visual Basic, PHP)
• the string "537"? (Java, Scala, Kotlin)
• a pointer? (C, C++)
• something else?

and what about

37 / "5"

?
if we now add two arbitrary expressions

e1 + e2

how can we decide whether this is legal and which operation to perform?

the answer is typing, a program analysis that binds types to each sub-expression, to rule out inconsistent programs
some languages are dynamically typed: types are bound to values and are used at runtime

examples: Lisp, PHP, Python, Julia

other languages are statically typed: types are bound to expressions and are used at compile time

examples: C, C++, Java, OCaml, Rust, Go
a language may use **both** static and dynamic typing

we will illustrate it with Java at the end of this lecture
1. static typing
   1.1 type checking WHILE
   1.2 type safety
2. implementing type checking
3. subtyping
4. overloading
static typing
well-typed programs do not go wrong
goals of typing

- type checking must be **decidable**

- type checking must reject programs whose evaluation would fail; this is **type safety**

- type checking must not reject too many non-absurd programs; the type system must be **expressive**
several solutions

1. any sub-expression is annotated with a type

```c
int f(int x) { int y = ((x:int)+(1:int):int); ... }
```

type checking is easy but this is unmanageable for the programmer

2. only annotate variable declarations (C, C++, Java, etc.)

```c
int f(int x) { int y = x+1; return y; }
```

3. only annotate function parameters (C++ 11, Java 10)

```c
int f(int x) { var y = x+1; return y; }
```

4. no annotation at all \(\Rightarrow\) **type inference** (OCaml, Haskell, etc.)

```ocaml
fun x -> x+1
```
let us consider the language \texttt{WHILE} from lecture 2

to make it more interesting, let us add \texttt{records}
(and any variable is a record)
e ::= expression
  | c integer or Boolean constant
  | x variable
  | e.f field access
  | e op e binary operator (+, <, ...)

s ::= statement
  | e.f ← e assignment
  | if e then s else s conditional
  | while e do s loop
  | s; s sequence
  | skip do nothing
\[ \begin{align*}
  x.a & \leftarrow 0; \\
  x.b & \leftarrow 1; \\
  \text{while } x.b < 100 \text{ do} \\
  & \quad x.b \leftarrow x.a + x.b; \\
  & \quad x.a \leftarrow x.b - x.a
\end{align*} \]
the notion of value from lecture 2 is updated

\[ \nu ::= \text{value} \]
\[ | \quad n \text{ integer value} \]
\[ | \quad b \text{ Boolean value} \]
\[ | \quad x \text{ address (here the name of the variable)} \]

we also update the environment \( E \),
which now maps pairs \((x, f)\) to values \( E(x, f) \)
we define a big-step operational semantics for expressions

\[ E, e \rightarrow v \]

and a small-step operational semantics for statements

\[ E, s \rightarrow E', s' \]
semantics of expressions

\[ E, n \rightarrow n \quad E, b \rightarrow b \]

\[ E, x \rightarrow x \]

\[ E, e \rightarrow x \quad (x, f) \in \text{dom}(E) \]
\[ E, e.f \rightarrow E(x, f) \]

\[ E, e_1 \rightarrow n_1 \quad E, e_2 \rightarrow n_2 \quad n = n_1 + n_2 \]
\[ E, e_1 + e_2 \rightarrow n \]

etc.
semantics of statements

\[ E, e_1 \rightarrow x \quad E, e_2 \rightarrow \nu \quad (x, f) \in \text{dom}(E) \]

\[ E, e_1.f \leftarrow e_2 \rightarrow E\{(x, f) \mapsto \nu\}, \text{skip} \]

\[ E, \text{skip}; s \rightarrow E, s \]

\[ E, e \rightarrow \text{true} \quad E, e \rightarrow \text{false} \]

\[ E, \text{if } e \text{ then } s_1 \text{ else } s_2 \rightarrow E, s_1 \]

\[ E, \text{while } e \text{ do } s \rightarrow E, s; \text{while } e \text{ do } s \]

\[ E, e \rightarrow \text{true} \quad E, e \rightarrow \text{false} \]

\[ E, \text{while } e \text{ do } s \rightarrow E, \text{skip} \]
we introduce **types**, with the following abstract syntax

\[
\tau ::= \text{type} | \text{int} \quad \text{type of integer values} \\
| \text{bool} \quad \text{type of Boolean values} \\
| \{ f : \tau ; \ldots ; f : \tau \} \quad \text{record type}
\]
the type of a variable is given by a **typing environment** \( \Gamma \) (a function from variables to types)

the **typing judgment** is written

\[
\Gamma \vdash e : \tau
\]

and reads “in typing environment \( \Gamma \), expression \( e \) has type \( \tau \)”

we use inference rules to define \( \Gamma \vdash e : \tau \)
typing expressions

\[ \Gamma \vdash n : \text{int} \qquad \Gamma \vdash b : \text{bool} \]

\[ x \in \text{dom}(\Gamma) \]
\[ \Gamma \vdash x : \Gamma(x) \]

\[ \Gamma \vdash e : \{ \ldots ; f : \tau ; \ldots \} \]
\[ \Gamma \vdash e.f : \tau \]

\[ \Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int} \]
\[ \Gamma \vdash e_1 + e_2 : \text{int} \quad \text{etc.} \]
with $\Gamma = \{ x \mapsto \{ a : \text{int}; b : \text{int} \} \}$, we have

$$
\begin{array}{c}
\Gamma \vdash x : \{ a : \text{int}; b : \text{int} \} \\
\hline
\Gamma \vdash x.a : \text{int} \quad \Gamma \vdash 1 : \text{int} \\
\hline
\Gamma \vdash x.a + 1 : \text{int}
\end{array}
$$

this derivation is a proof that $x.a+1$ is well-typed
expressions without a type

in the same environment, we cannot type expressions such as

\[ x.c \]

or

\[ 42.a \]

or

\[ 1 + \text{true} \]

this is precisely what we want, for these expressions have no value in our semantics.
to type statements, we introduce a new judgment

\[ \Gamma \vdash s \]

that reads “in environment \( \Gamma \), statement \( s \) is well-typed”
type checking statements

\[\Gamma \vdash \text{skip} \quad \frac{\Gamma \vdash s_1 \quad \Gamma \vdash s_2}{\Gamma \vdash s_1; s_2}\]

\[\frac{\Gamma \vdash e_1 : \{\ldots; f : \tau : \ldots\} \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1.f \leftarrow e_2}\]

\[\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash s_1 \quad \Gamma \vdash s_2}{\Gamma \vdash \text{if } e \text{ then } s_1 \text{ else } s_2}\]

\[\frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash s}{\Gamma \vdash \text{while } e \text{ do } s}\]
well-typed programs do not go wrong
let us show that our type system is safe wrt our operational semantics

**Theorem (type safety)**

\[
\text{If } \Gamma \vdash s, \text{ then the reduction } s \text{ is either infinite or reaches } \text{skip}.
\]

or, equivalently,

**Theorem**

\[
\text{If } \Gamma \vdash s \text{ and } E, s \rightarrow^* E', s' \text{ and } s' \text{ is irreducible, then } s' \text{ is skip.}
\]
this means evaluation won’t be **stuck** or any expression such as

\[ 42.a \]

or on a statement

\[ \text{if } e \text{ then } s_1 \text{ else } s_2 \]

where \( e \) does not evaluate to either true or false
let us show first that well-typed expressions do evaluate successfully

\[ \text{if } \Gamma \vdash e : \tau, \text{ then } E, e \rightarrow v \]

stated as such, this is not correct: we need a relationship between \( \Gamma \) and \( E \)

counterexample:

\[
\begin{align*}
\Gamma &= \{ x \mapsto \{ a : \text{int} \} \} \\
e &= x.a \\
E &= \emptyset
\end{align*}
\]
consistency of environments

Definition (well-typed environment)

An execution environment $E$ is well-typed in a typing environment $\Gamma$, written $\Gamma \vdash E$, if

$$\forall x, \text{ if } \Gamma(x) = \{ \ldots f : \tau \ldots \} \text{ then } (x, f) \in \text{dom}(E) \text{ and } \Gamma \vdash E(x, f) : \tau.$$
Lemma (evaluation of a well-typed expression)

If $\Gamma \vdash e : \tau$ and $\Gamma \vdash E$, then $E, e \rightarrow v$ and $\Gamma \vdash v : \tau$.

proof: by induction on the derivation $\Gamma \vdash e : \tau$.

- $e = c$ immediate with $v = c$
- $e = x$ immediate with $v = x$
- $e = e_1.f$ by IH $E, e_1 \rightarrow v_1$ and $\Gamma \vdash v_1 : \tau_1$ with $\tau_1 = \{ \ldots f : \tau \ldots \}$.
  so $v_1$ is a variable $x$ and $v = E(x, f)$
  since $E$ is well-typed, we have $\Gamma \vdash v : \tau$

- $e = e_1 + e_2$ by IH on $e_1$ and $e_2$ we have $E, e_i \rightarrow v_i$ and $\Gamma \vdash v_i : \text{int}$,
  so $v_1$ and $v_2$ are integers and we conclude with $v = v_1 + v_2$

$\square$
the type safety proof is based on two lemmas

**Lemma (progress)**

If \( \Gamma \vdash s \) and \( \Gamma \vdash E \), then either \( s \) is \( \text{skip} \), or \( E, s \rightarrow E', s' \).

**Lemma (preservation)**

If \( \Gamma \vdash s \), if \( \Gamma \vdash E \) and if \( E, s \rightarrow E', s' \) then \( \Gamma \vdash s' \) and \( \Gamma \vdash E' \).
Lemma (progress)

If $\Gamma \vdash s$ and $\Gamma \vdash E$, then either $s$ is $\text{skip}$, or $E, s \rightarrow E', s'$.

proof: by induction on the derivation $\Gamma \vdash s$

$s = \text{skip}$ immediate

$s = s_1; s_2$ if $s_1 = \text{skip}$, we have $E, s_1; s_2 \rightarrow E, s_2$
 otherwise, we use IH on $s_1$, so $E, s_1 \rightarrow E', s'_1$ and thus $E, s_1; s_2 \rightarrow E', s'_1; s_2$

$s = e_1.f \leftarrow e_2$ since $e_1$ and $e_2$ are well-typed, they evaluate to $x$ and $\nu$ respectively

since $\Gamma \vdash x : \{\ldots f : \tau \ldots\}$ we have $(x, f) \in \text{dom}(E)$ and thus $E, s \rightarrow E', \text{skip}$ with $E' = E\{(x, f) \mapsto \nu\}$

other cases left as exercise
then we show

**Lemma (preservation)**

If $\Gamma \vdash s$, if $\Gamma \vdash E$ and if $E, s \rightarrow E', s'$ then $\Gamma \vdash s'$ and $\Gamma \vdash E'$.

**proof:** by induction on the derivation $\Gamma \vdash s$

$s = s_1; s_2$ we have $\Gamma \vdash s_1$ and $\Gamma \vdash s_2$

- if $s_1 = \text{skip}$, then $E, s_1; s_2 \rightarrow E, s_2$
- otherwise, $E, s_1 \rightarrow E', s'_1$ and by IH $\Gamma \vdash s'_1$ and $\Gamma \vdash E'$
  so $\Gamma \vdash s'_1; s_2$

$s = e_1.f \leftarrow e_2$ we have $E, e_1 \rightarrow x$ and $E, e_2 \rightarrow v$ and $s' = \text{skip}$ (so $\Gamma \vdash s'$) and $E' = E\{(x, f) \mapsto v\}$

but $\Gamma \vdash e_1 : \{\ldots f : \tau \ldots\}$ and $\Gamma \vdash e_2 : \tau$ so $\Gamma \vdash v : \tau$ (see slide ??) and thus $\Gamma \vdash E'$

other cases left as exercise  □
now we can deduce type safety easily

**Theorem (type safety)**

If $\Gamma \vdash s$ and $E, s \rightarrow^* E', s'$ and $s'$ is irreducible, then $s'$ is skip.

**proof:** we have $E, s \rightarrow E_1, s_1 \rightarrow \cdots \rightarrow E', s'$ and by repeated applications of the preservation lemma, we have $\Gamma \vdash s'$ by the progress lemma, $s'$ is reducible or is skip so this is skip $\square$
the lecture notes contains a similar proof for Mini-ML, with types as follows

\[
\tau ::= \text{int} | \text{bool} | \ldots \quad \text{base types} \\
| \tau \to \tau \quad \text{function type} \\
| \tau \times \tau \quad \text{type of a pair}
\]

as we just did, the proof is based on progress and preservation properties

see chapter 5
languages such as Java or OCaml enjoy such a type safety property which means that the evaluation of an expression of type $\tau$

- either does not terminate
- or raises an exception
- or terminates on a value with type $\tau$

in OCaml, the absence of `null` makes it a rather strong property
implementing type checking
implementing type checking

there is a difference between the typing rules, which define the relation

\[ \Gamma \vdash e : \tau \]

and the type checking algorithm, which checks that a given expression \( e \) is well-typed in some environment \( \Gamma \)

for instance

- the type \( \tau \) is not necessarily given (type inference)
- several rules may apply for a single construct
- an expression may have several types
in our case

the case of **WHILE** is simple, as a single rule applies for each expression

we say that typing is **syntax-directed**

the type checking is then implemented with a linear time traversal of the program
practical considerations

• we do not simply say *type error*
  
  but we **explain** the type error precisely

• we **keep** types for the further phases of the compiler
to do this, we **decorate** abstract syntax trees

- **input** of type checking contains positions in source code
- **output** of type checking contains types

![Diagram](syntax_trees_with_locations → type_checker → syntax_trees_with_types)
decorated AST

in OCaml

```ocaml
type loc = ...

type expr =
| Evar of string
| Econst of int
| Efield of expr * string
...
```

in Java

```java
class Loc { ... }

abstract class Expr {

}

class Evar extends Expr {...}
class Econst extends Expr {...}
class Efield extends Expr {...}
...
```
in OCaml

```ocaml
type loc = ...

type expr = {
    desc: desc;
    loc : loc;
}

and desc =
| Evar of string
| Econst of int
| Efield of expr * string
...
```

in Java

```java
class Loc { ... }

abstract class Expr {
    Loc loc;
}

class Evar extends Expr {...}
class Econst extends Expr {...}
class Efield extends Expr {...}
...
```
we signal a type error with an exception

the exception contains

• a message explaining the error
• a position in the source code
we catch this exception in the main function

we display the position and the message

test.c:8:14: error: too few arguments to function 'f'
we set up an abstract syntax for types

type typ = ...
class Typ { ... }

and a new abstract syntax for programs

```plaintext
type texpr = {
    tdesc: tdesc;
    typ : typ
}
and tdesc =
| Tvar of string
| Tconst of int
| Tfield of texpr * string
...
```

abstract class Texpr {
    Typ typ;
}

class Tvar extends Texpr {...}
class Tconst extends Texpr {...}
class Tfield extends Texpr {...}
...
the type checker turns a parsed syntax tree into another, typed syntax tree

yet this is efficient, since

- it is typically a linear traversal
- former AST are collected by the GC
subtyping
we say that a type $\tau_1$ is a **subtype** of a type $\tau_2$, which we write

$$\tau_1 \leq \tau_2$$

if any value with type $\tau_1$ can be considered as a value with type $\tau_2$
in many languages, there is subtyping between numerical types

in Java, it is as shown on the right

thus we can write

```java
int n = 'a';
```

but not

```java
byte b = 144;
```
in an object-oriented language, inheritance induces subtyping: if a class B inherits from a class A, we have

\[ B \leq A \]

i.e. any value of type B can be seen as a value of type A
the two classes

```java
class Vehicle { ... void move() { ... } ... }
class Car extends Vehicle { ... void move() { ... } ... }
```

induce the subtyping relation

\[
\text{Car} \leq \text{Vehicle}
\]

and thus we can write

```java
Vehicle v = new Car();
v.move();
```
the construct `new C(...)` builds an object of class `C`, and the class of this object cannot be changed in the future; this is the dynamic type of the object.

however, the static type of an expression, as computed by the compiler, may differ from the dynamic type, because of subtyping.

when we write

```java
Vehicle v = new Car();
v.move();
```

variable `v` has type `Vehicle`, but the method `move` that is called is that of class `Car` (we’ll explain how in another lecture)
in many cases, the compiler **cannot** determine the dynamic type

example:

```java
void moveAll(LinkedList<Vehicule> l) {
    for (Vehicule v: l)
        v.move();
}
```
sometimes we need to force the compiler’s hand, which means we claim that a value has some type

we call this **type casting** (or simply cast)

Java’s notation, inherited from C, is

\[(\tau)e\]

the static type of this expression \(\tau\)
using a cast, we can write

```java
int n = ...;
byte b = (byte)n;
```

in this case, there is no dynamic verification (if the integer is too large, it is truncated)
let us consider

\[(C)e\]

where

- \(D\) is the dynamic type of (the object designated by) \(e\)
- \(E\) is the static type of expression \(e\)

does not apply.

there are three cases

- \(C\) is a super class of \(E\): this is an upcast and the code for \((C)e\) is that of \(e\) (but the cast has some influence anyway, since \((C)e\) has type \(C\))
- \(C\) is a subclass of \(E\): this is a downcast and the code contains dynamic test to check that \(D\) is indeed a subclass of \(C\)
- \(C\) is neither a subclass nor a super of \(E\): the compiler rejects the program with a type error
class A {
    int x = 1;
}

class B extends A {
    int x = 2;
}

B b = new B();
System.out.println(b.x); // 2
System.out.println(((A)b).x); // 1
void m(Vehicle v, Vehicle w) {
    ((Car)v).await(w);
}

nothing guarantees that the object passed to m will be a car; in particular, it could have no method await!

the dynamic test is required

Java raises ClassCastException is the test fails
testing subtyping dynamically

to allow defensive programming, there exists a Boolean construct

\[ e \text{ instanceof } C \]

that checks whether the class of \( e \) is indeed a subclass of \( C \)

it is idiomatic to do

```java
if (e instanceof C) {
    C c = (C)e;
    ...
}
```

in this case, the compiler makes an optimization to perform a single test
overloading

(surcharge en français)
overloading is the ability to reuse the same name of several operations

overloading is handled at compile time, using the number and the (static) types of arguments

caveat: not to be confused with overriding (see lecture 6)
in Java, operation + is overloaded

```java
int n = 40 + 2;
String s = "foo" + "bar";
String t = "foo" + 42;
```

these are three distinct operations

```java
int +(int, int)
String +(String, String)
String +(String, int)
```
when we write

```java
int n = 'a' + 42;
```

this is subtyping that allows us to consider 'a' with type char as a value of type int, and thus the operation is +(int, int)

but when we write

```java
String t = "foo" + 42;
```

this is not subtyping (int \not\subseteq String)

in particular, we cannot write

```java
String t = 42;
```
in Java, one cannot overload operators such as +
but one can overload methods/constructors

```java
int f(int n, int m) { ... }
int f(int n) { ... }
int f(String s) { ... }
```
this is exactly as if we had written

```c
int f_int_int(int n, int m) { ... }
int f_int (int n) { ... }
int f_String (String s) { ... }
```

the compiler uses the static types of $f$’s arguments to determine which method to call
yet overloading resolution can be tricky

```java
class A {...}
class B extends A {
    void m(A a) {...}
    void m(B b) {...}
}
```

with

```java
{ ... B b = new B(); b.m(b); ... }
```

both methods apply

this is method m(B b) that is called, because it is considered more precise
some cases are ambiguous

class A {...}
class B extends A {
    void m(A a, B b) {...}
    void m(B b, A a) {...}
}
{ ... B b = new B(); b.m(b, b); ... }

and reported as such

test.java:13: reference to m is ambiguous,
    both method m(A,B) in B and method m(B,A) in B match
Java’s overloading resolution

to each method defined in class C

$$\tau \ m(\tau_1 \ x_1, \ldots, \tau_n \ x_n)$$

we set the profile \((C, \tau_1, \ldots, \tau_n)\)

then we order profiles: \((\tau_0, \tau_1, \ldots, \tau_n) \sqsubseteq (\tau'_0, \tau'_1, \ldots, \tau'_n)\) if and only if \(\tau_i\) is a subtype of \(\tau'_i\) for all \(i\)

for a call

$$e.m(e_1, \ldots, e_n)$$

where \(e\) has static type \(\tau_0\) and \(e_i\) has static type \(\tau_i\), we consider the set of all minimal elements in the set of all compatible profiles

- no element ⇒ no method applies
- several elements ⇒ ambiguity
- a single element ⇒ this is the method to call
• lab 4 = we start the project
  • static typing of Mini Python
  • (the parser is provided)

• next lecture
  • evaluation strategy
  • parameter passing