INF 555

Geometric Modeling: Digital Representation and Analysis of Shapes

TD 5: Subdivision surfaces

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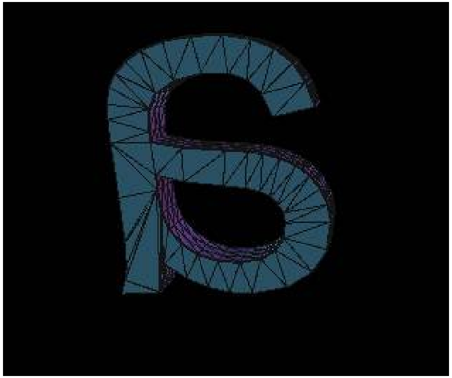
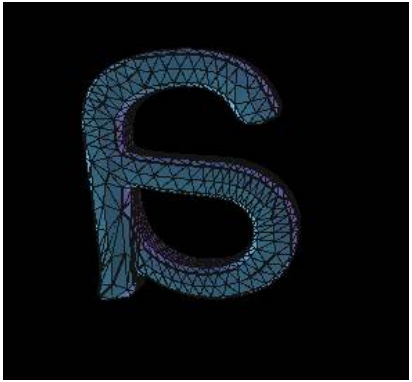
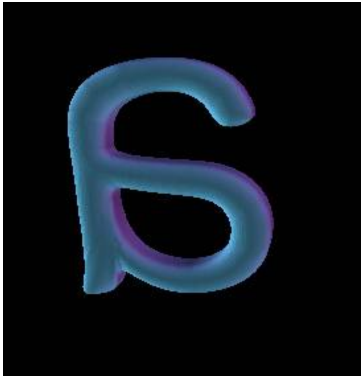
Loop subdivision scheme



276 vertices (original model)



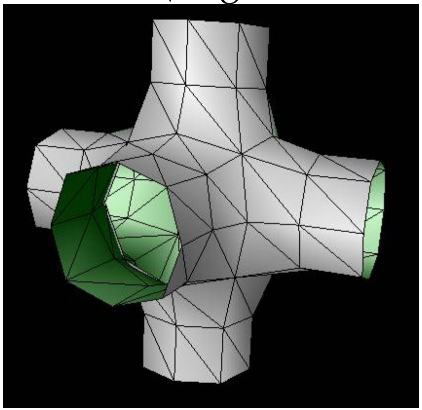
1104 vertices (after one iteration)

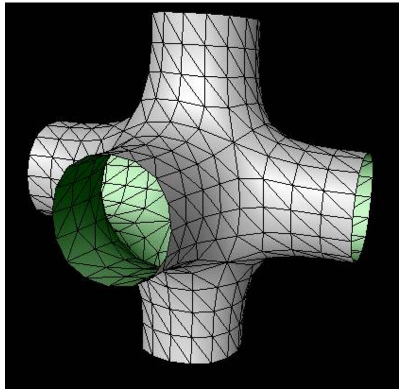


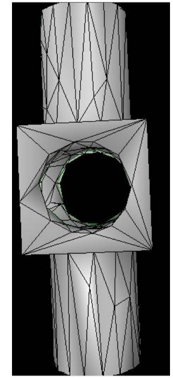
17664 vertices (after 3 iterations)

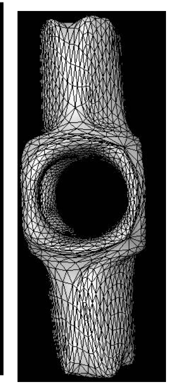
Loop subdivision: triangle meshes

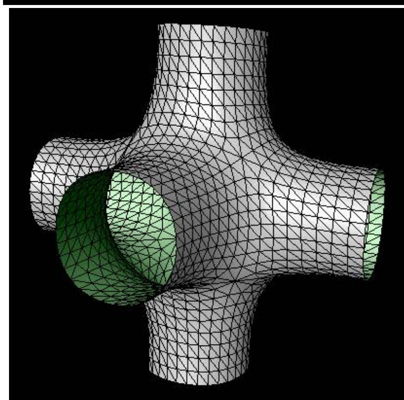
32 vertices (original model)

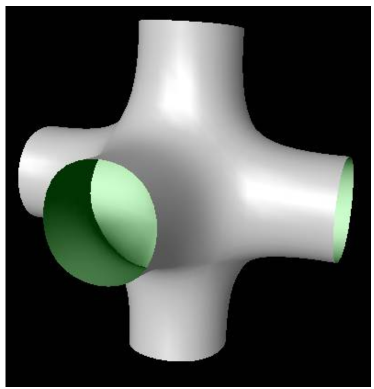






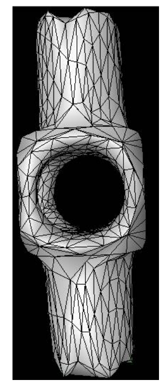


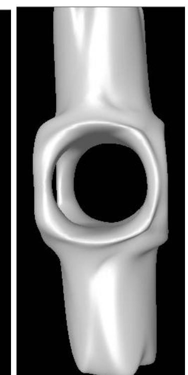




1628 vertices

3 iterations of
Loop subdivision scheme

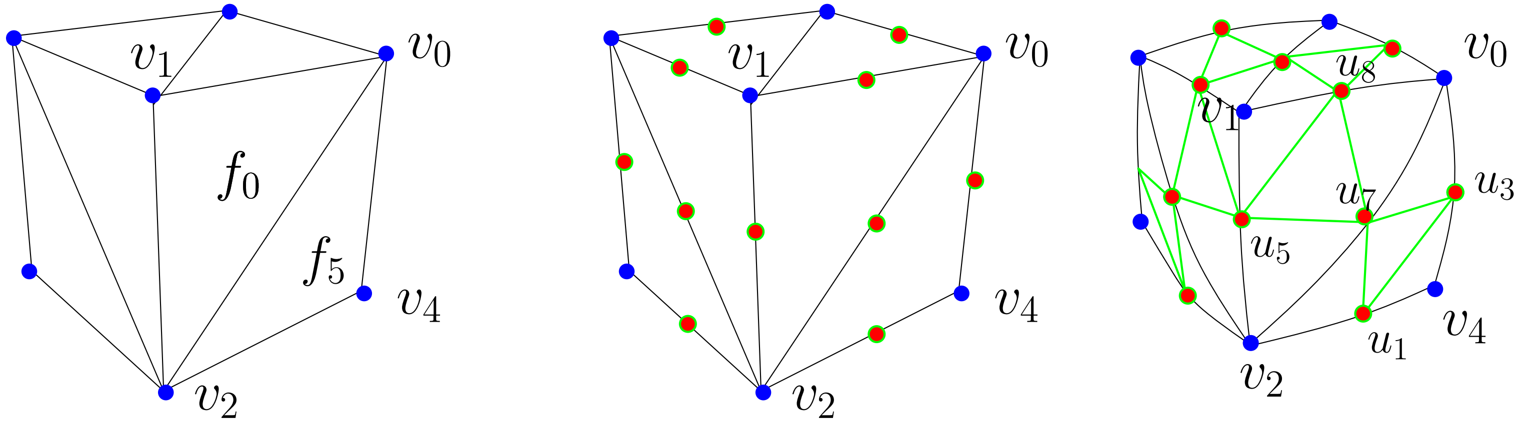




Loop subdivision: updating the connectivity

split all edges, by inserting a midpoint

subdivide each face into 4 triangles



n **vertices,** e **edges,** f **faces**

$n+e$ **vertices,** $4f$ **faces**

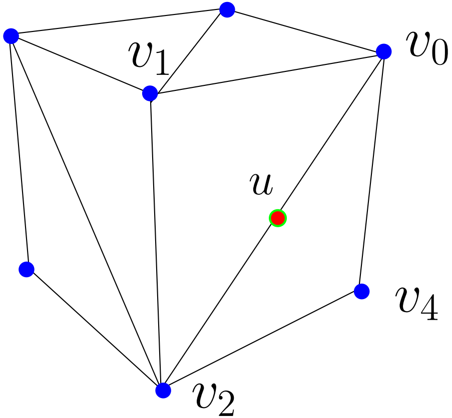
$$f_0 : v_0, v_1, v_2$$
$$\ldots$$
$$f_5 : v_2, v_4, v_0$$
$$\ldots$$

$f_0 \rightarrow (f_0', f_1', f_2', f_3')$

$$f_0' : u_7, u_8, u_5$$
$$\ldots$$
$$f_2' : u_7, u_5, v_2$$
$$\ldots$$

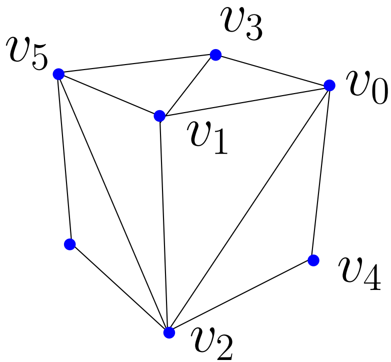
Loop subdivision: updating the geometry

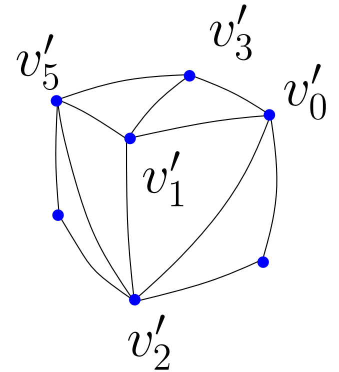
First compute edge points u_k



$$u = \frac{3}{8}v_0 + \frac{3}{8}v_2 + \frac{1}{8}v_1 + \frac{1}{8}v_4$$

Compute new locations v_i' of initial vertices





$$v_i' = (1 - \alpha d)v_i + \alpha \sum_{j=1}^{d} v_{i_j}$$

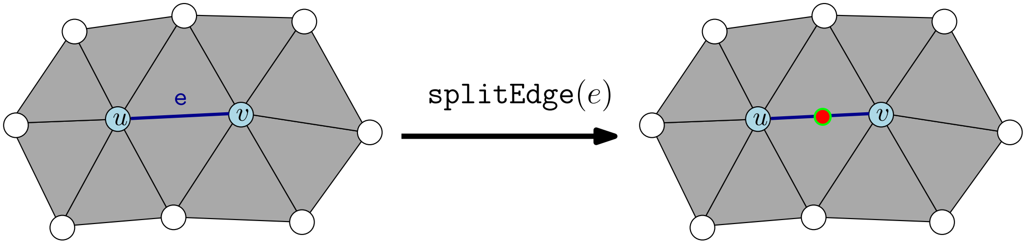
d is the *degree* of vertex v_i
v_{i_j} is the j-th neighbor of v_i

$\alpha = \frac{3}{16}$, if $d = 3$

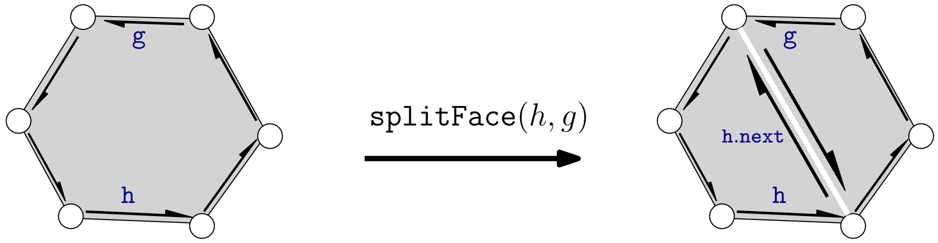
$\alpha = \frac{3}{8d}$, if $d > 3$

"Topological changes": overview

edge splitting (insert a new vertex)



face splitting (create a new face, inserting two new halfedges)

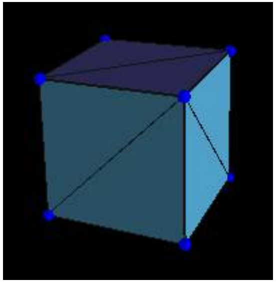


`h`, g must belong to the same face

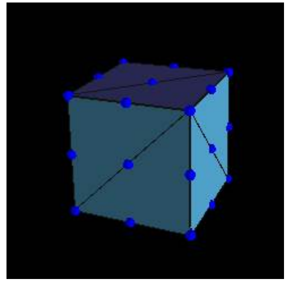
`splitFace`(h, g) returns as result the halfedge `h.next`

Implementing Loop subdivision: suggestions

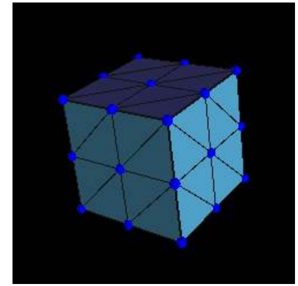
edge splitting (insert a new vertex)



perform `splitEdge`(e)

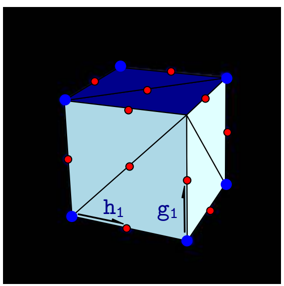


subdivide each face

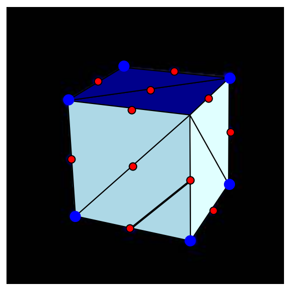


subdivide faces using `splitFace`() operator

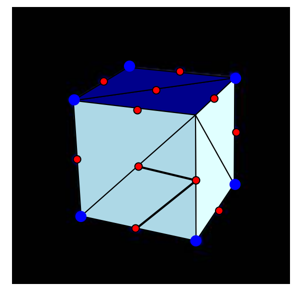
(faces are all of degree $3 + 3$)



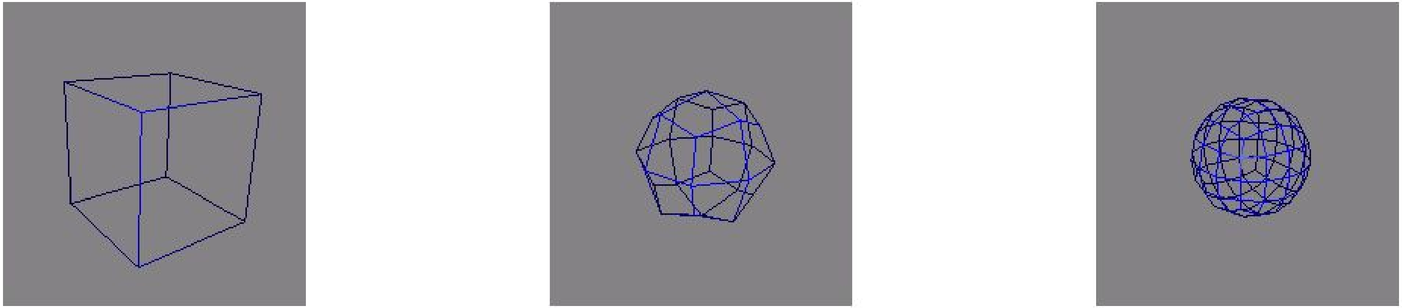
`splitEdge`(h_1, g_1)



`splitEdge`$(\ldots)$



Catmull-Clark subdivision scheme

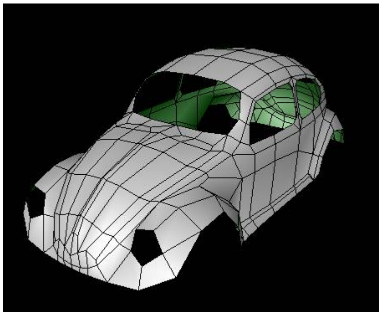


Cube, smoothed twice with repeated applications of the Catmull-Clark scheme

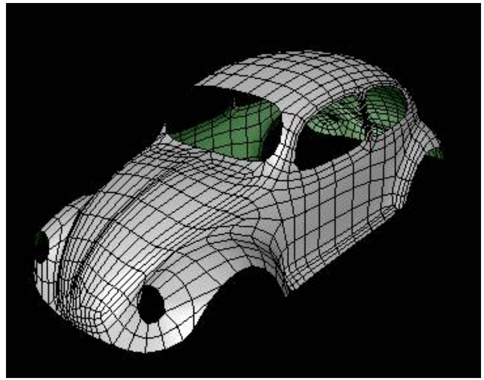
Catmull-Clark subdivision for arbitrary meshes

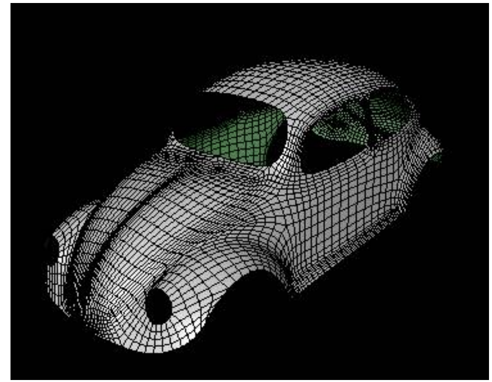
Catmull-Clark subdivision (1978)

arbitrary polygonal mesh

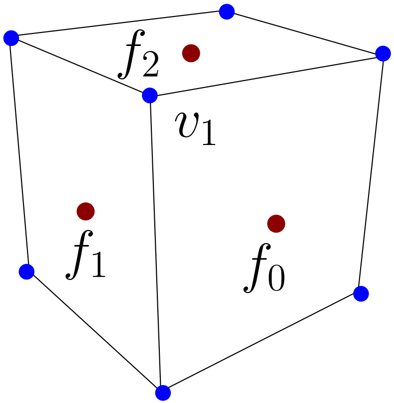


quad mesh

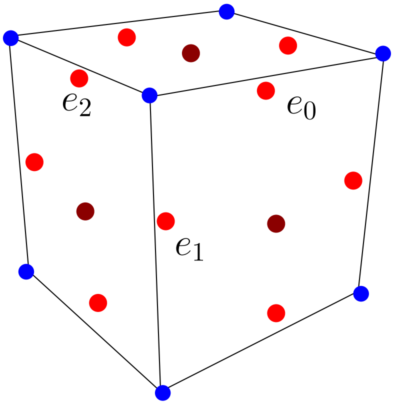




Catmull-Clark: updating the connectivity

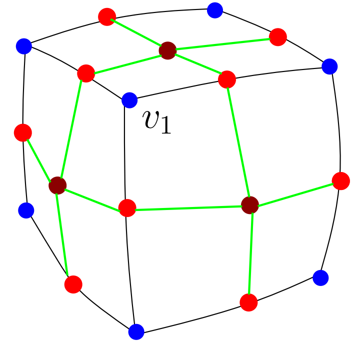


compute *centroids* (for each face)



split all edges, by inserting an edge point

remarks: the location of edge points do not coincide with the midpoint (see next slide)



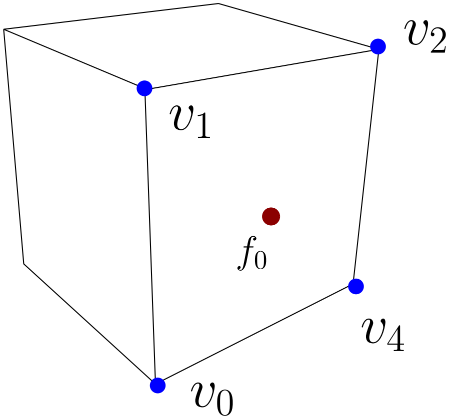
quadrangulate each face

add face (v_1, e_1, c_0, e_0)

n vertices, e edges, f faces

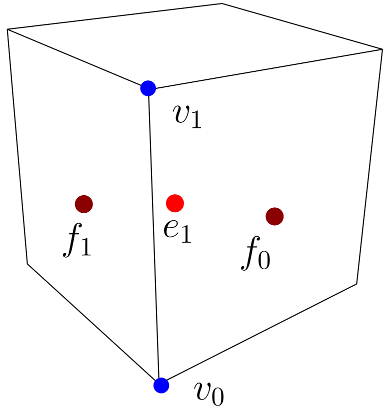
$n+e+f$ vertices, $2e$ faces

Catmull-Clark: updating the geometry



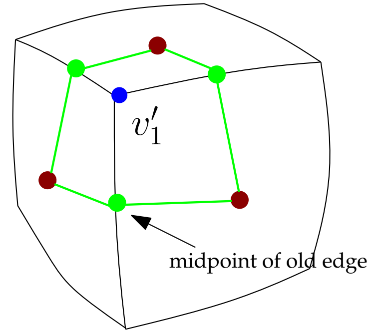
compute *centroids* (for each face)

$$f_i = \frac{1}{d}\sum_{j=1}^{d} v_{i_j}$$



split all edges, by inserting an *edge point*

$$e_1 = \frac{1}{4}v_0 + \frac{1}{4}v_1 + \frac{1}{4}f_1 + \frac{1}{4}f_0$$



quadrangulate each face

add face (v_1, e_1, c_0, e_0)

compute a weighted average:

$$v_{new} = \frac{(d-3)v_{old}+2E+F}{d}$$

E := average of *midpoints* of old edges incident to the vertex

F := average of *face points* (corresponding to faces incident to the vertex)

Implementing Catmull-Clark: suggestions

iteratively add *quadrangular faces*