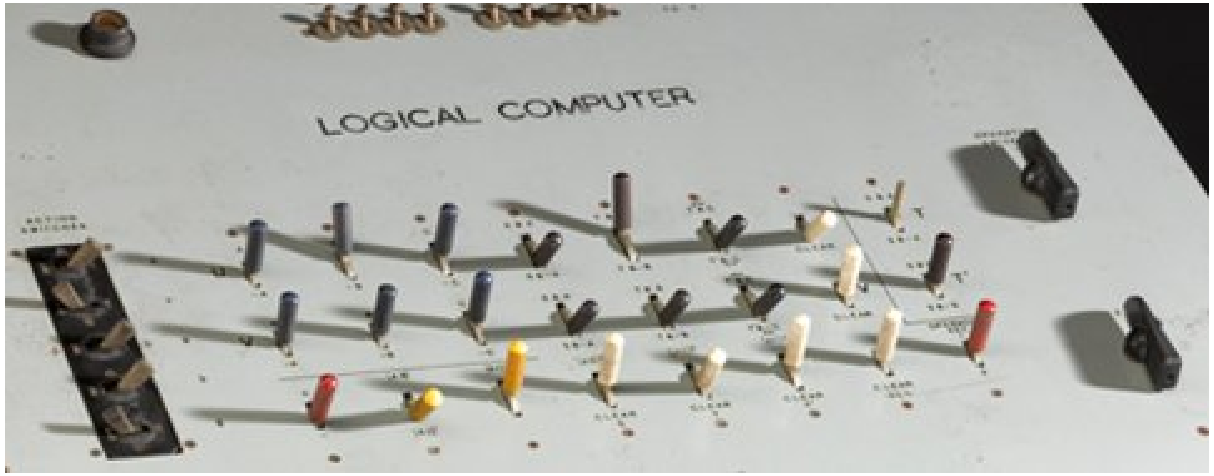
INF551

Computational Logic:

Artificial Intelligence in Mathematical Reasoning



Stéphane Graham-Lengrand

Stephane.Lengrand@Polytechnique.edu

Practical information

Timetable:

9 (lectures & practicals) Fridays [14:00-16:00] & [16:15-18:15], from 22nd September

Room: PC18

T.A.: Roberto Blanco, `roberto.blanco@inria.fr`

Website (slides, links to practical sheets, . . .):

http://www.enseignement.polytechnique.fr/informatique/INF551/

Evaluation:

25%: Class participation (once in the 8 remaining weeks: showing to the class what you have done / tried to do from one week to the next)

75%: Exam on 22nd December (using paper+laptop)

. . . or, for some motivated students, mini-project in connection to the course (suggestions are online), must be convinced you also know the rest of the course

Course notes

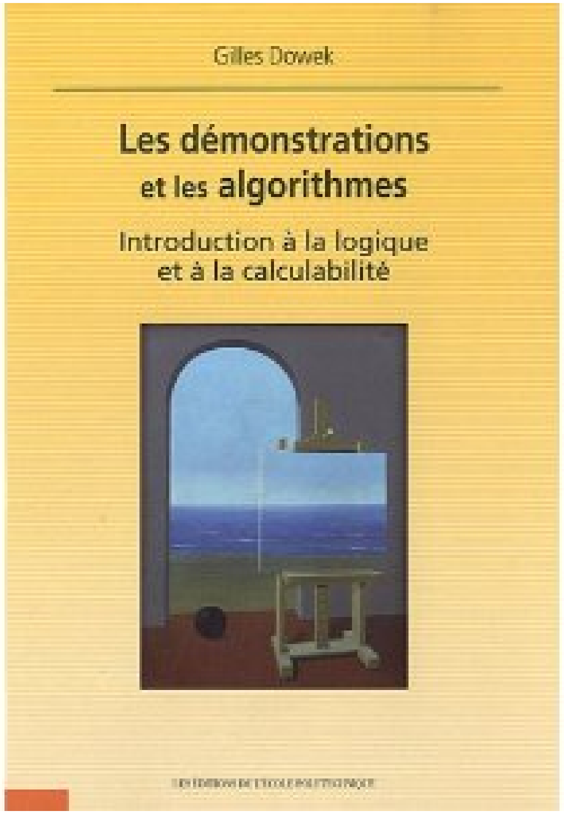
This is a half-old / half-new course.

Previous version

= G. Dowek's book

= distributed course notes

Available in English!



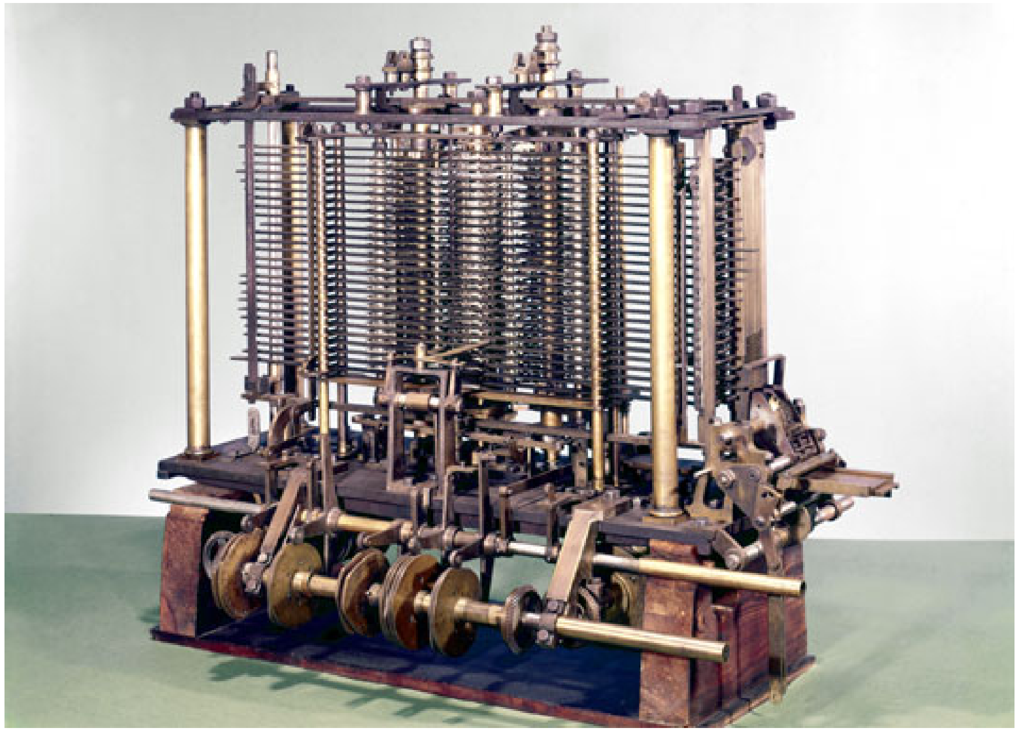
Version now affected by introduction of INF412 in undergrad. curriculum

This year in INF551 we shall:

- treat parts of book not treated in undergrad. curriculum
- develop other parts further
- make TD more practical / machine-based.

Lecture 0

Introduction



Reasoning and computing

You reason since the first words

You compute since kindergarten

You encountered the definition of such mechanisms at undergrad. level (INF412)

Similar situation in the History of Science:

reasoning and computing since Stone Age

. . . properly defined during XIXth or XXth century

Reasoning vs. computing

Easier to compute than to reason (especially since computers)

Follow some computing rules.

No intelligence needed, just time and space.

Can be automated (done by a machine).

Question addressed by this course:

Can we use computing to reason?

What do we mean by “reasoning”?

“The art of establishing truth.”

Since ancient times, the question is:

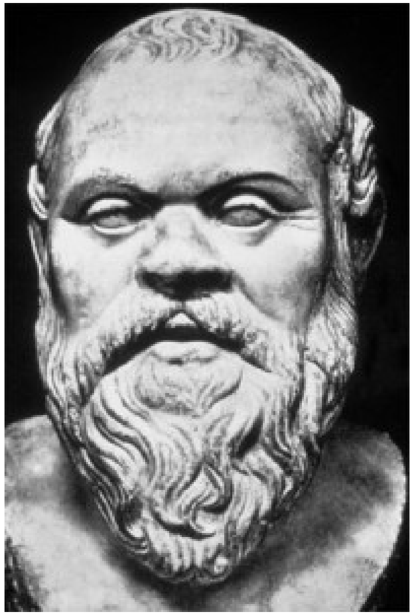
Is a particular mathematical statement true or false?

What do we mean by “true”?

Since ancient times: confronting with reality

First mathematical fields = arithmetic and geometry

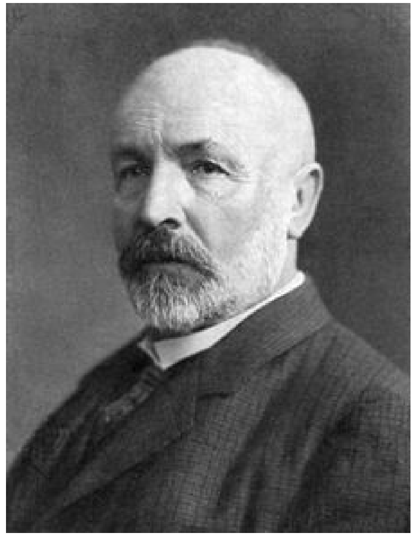
c.f. Plato (e.g. Meno)



Since then, the fields of mathematics have...

- diversified
- got further and further away from witnessable reality

XIXth century: Logic crisis



Most notable example: The notion of *infinity*

How can truth be checked against reality?

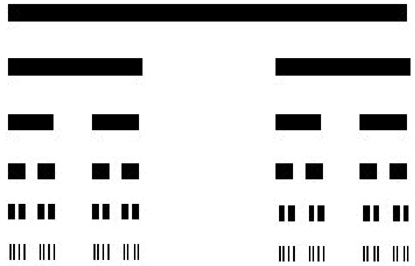
Triggered by works of Cantor identifying $\neq$ notions of infinity

$\mathbb{R} \simeq$ converging sequences in $\mathbb{Q}$ (1870-1872)

Bijections & $|\mathbb{Q}| \neq |\mathbb{R}|$ (1874), $|\mathcal{P}(A)| > |A|$, $|\mathbb{R}| = \left|\mathbb{R}^2\right|$

to Dedekind in 1877: “Je le vois, mais je ne le crois pas!”

Continuum Hypothesis (1878)



1874-1884: developped ideas into theory of ordinals and cardinals

1883: Cantor’s set premisses of Mandelbrot’s fractal theory

Huge impact on topology & measure theory (Borel-Lebesgue,. . .)

Criticised by e.g. Kronecker

for not being able to produce sets in finite number of “steps” from natural numbers

First ideas of constructivism. To Lindemann proving that π is transcendental:

“Why study such problems when irrational numbers do not exist?”

Fear of Axioms, Fear of paradoxes/inconsistencies (lots found between 1885-1905)

A standard example

Suppose a and b are strictly positive integers.

$$
\begin{aligned}
a &= b \\
a^2 &= ab \\
a^2 - b^2 &= ab - b^2 \\
(a-b)(a+b) &= b(a-b) \\
a + b &= b \\
b + b &= b \\
2b &= b \\
2 &= 1
\end{aligned}
$$

Teacher: “This is wrong! You divide by 0!”

Pupil: “Can I not?”

Reasoning is about rules (applying them correctly), more than about truth.

$\Rightarrow$ **Computation?**

Computation & the foundations of Mathematics

From the 1870-1940 research period: *rules of logical inference* formalised

Combined with *axioms* to form the notion of *proof*, they

- shifted the problem of truth to the problem of provability:

  Is a statement true or false? $\Rightarrow$ Is a statement provable or not?

- suggested that rigourous reasoning could be reduced to a mechanical task:

"*If controversies were to arise, there would be no more need of disputation between two philosophers than between two calculators. For it would suffice for them to take their pencils in their hands and to sit down at the abacus, and say to each other: Let us calculate.*"

Leibniz (1677)

Inference rules & mathematical axioms: incredibly tied together

"more-or-less" interchangeable, but rules are usually more "computer-friendly"

More generally: Computation intrinsically tied to the foundations of mathematics

(Which axioms & rules can be used to develop all mathematics

& describe any problem that we would like a machine to solve?)

Logic gave birth to computers and A.I.

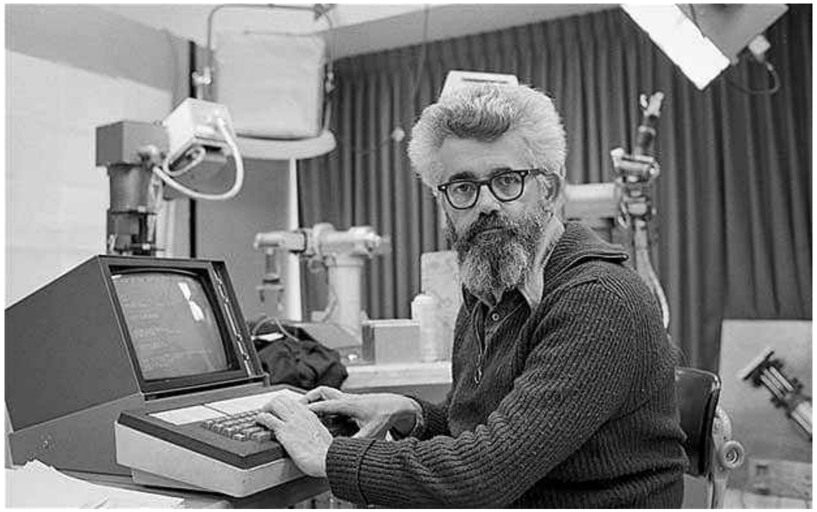
Computers invented on paper (30s) before invented in real life!

. . . for the very purpose of automating mathematical reasoning

. . . by the very people who cleared up the foundations of mathematics

(Goedel, Church, Turing)

“I propose to consider the question, ‘Can machines think?’ ” (1950)



“Artificial Intelligence”

coined by John McCarthy

at Dartmouth Conference (1956)

A very brief & approximate overview of A.I.

"Can machines think?"

Goals ranging from	Deduction, reasoning, problem solving	to	Interaction with human / environment, NLP

McCarthy & Minsky found A.I. lab at MIT (1958)

then McCarthy found Stanford A.I. lab (SAIL) (1963)

Approaches:	Symbolic A.I. Logic-based A.I.	Neural nets Cognitive simul.
	"The neats"	"The scruffies"
	(McCarthy, championing mathematical logic for A.I.)	(Minsky)
	Logic programming	Probabilistic methods
	Automated reasoning	Uncertain reasoning
	Constraint solving	Machine learning

In this course: Logic-based approach to A.I.

Several algorithmics disciplines related to proofs:

Inference rules

Algorithmics of Proof-checking	Algorithmics of Proof-searching	Algorithmics of Proof-reducing
Easy	Lect. 2-5	Lect. 9

Two questions remain central:

- Termination of such algorithms
- Determinism of such algorithms and computational cost

Some unexpected results: While proof-search can be implemented (under minimalistic hypotheses) as computational process, the converse is true:

All computation can be seen as the search for a mathematical proof

Proof-search not just 1 small algorithmic domain $\Rightarrow$ ubiquitous in CS

The plan

Week 1: Introduction and review of undergrad. material

Week 2,3,4,5: Algorithmics of proof-search

Week 6,7,8: Modelling all mathematical problems in universal framework

Week 9: Constructivism

Questions?